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Market structure, factor endowment, and technology adoption ☆

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ABSTRACT

This paper investigates how technology adoption depends on factor endowment when new, capital-intensive technology is privately accessible. The non-competitive market structure is shown to indirectly distort factor prices in general equilibrium, resulting in a nonmonotonic capital endowment impact on static allocation efficiency and the dynamic pattern of industrial upgrading. Moreover, an increase in the initial capital endowment may delay rather than facilitate the adoption of capital-intensive technology. Private accessibility to the new technology may also result in premature adoption, overutilization, and multiple equilibria. Welfare-enhancing policies are discussed.

1. Introduction

Economic growth is a process of industrial upgrading along which capital accumulates and technologies advance. Comin and Hobijn (2010) use cross-country data to show that, on average, it takes 45 years to adopt a new technology after it is invented. Such delays are even more pronounced in developing countries, so a fundamental question is what prevents poor countries from adopting better foreign technology (Parente and Prescott (1994, 1999)). The primary goal of this paper is to study how a potential investor (firm), who for some exogenous reasons has access to a new technology, decides whether and when to adopt this technology in a developing country. This potential investor could be a domestic entrepreneur who has learned the new technology abroad. In this paper, we focus on the technology adoption problem, so we take the existence of new technologies as exogenous.

To this end, we develop a simple dynamic general equilibrium model to (1) theoretically investigate how an existing but privately accessible capital-intensive technology is adopted in a two-factor market economy with endogenous saving and (2) analyze the economic efficiency associated with technology adoption. The model has one final good that can be produced by two alternative technologies, new and old. The two technologies differ in two dimensions. First, the new technology is more capital-intensive. Second, it is private to one potential entrant and subject to free imitation one period after adoption, whereas the old technology is publicly and freely accessible. Consequently, which technology is better depends on relative factor prices. Since the market structure for the final good could be endogenously noncompetitive, the factor prices might be indirectly distorted in a general-equilibrium fashion even if factor markets *per se* are perfect. In that case, market equilibrium factor prices may no longer serve as accurate signals to guide socially optimal technology adoption.

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A novel feature of this technology adoption model is that the monopolist of the new technology (hereafter called firm M) makes adoption and production decisions by considering the general equilibrium effect of its monopoly power on the intratemporal and intertemporal prices of production factors. The magnitude of the monopoly rent depends on the factor endowment, especially capital endowment, accumulated endogenously through households' savings. Households have rational expectations about the timing of the adoption of new technology, which in turn affects the market structure and the return to private technology. Once implemented, the new technology is fully imitated in the next period and becomes publicly available afterward, so the market structure restores perfect competition. Firm M understands that its adoption decision not only affects the dynamics of market structure (due to the lagged information externality) and current profit but also affects the intertemporal interest rate (that is, the time discounting rate of profit) due to the consumption-smoothing motive of forward-looking households. To make optimal decisions, firm M must account for all the aforementioned forces when pondering the following tradeoff: early implementation avoids time discounting of profit, but instantaneous profit can be larger in the future when the rental–wage ratio goes down due to endogenous capital accumulation.

We first study this problem in a static general equilibrium model, in which we characterize and compare two different cases. One is the first-best benchmark case when both technologies are publicly and freely available, so the market structure is perfect competition. The other is when the capital-intensive technology is privately known only to firm M, so firm M may have monopoly power subject to the limit pricing constraint because the old (labor-intensive) technology is publicly available. In both cases, the capital-intensive technology is adopted if and only if the capital endowment is sufficiently large. We show that monopoly still achieves the social (Pareto) efficiency when capital is sufficiently abundant, which appears to contradict the textbook partial-equilibrium result. To resolve this puzzle, observe that the monopoly structure has two opposite effects in general equilibrium. One is the conventional negative price effect on demand due to the price markup. Conversely, the other is the positive income effect on demand, as monopoly profit now becomes part of household income. It turns out that these two forces cancel out exactly when the capital endowment is sufficiently large. In other words, the key difference is that the demand function for the final good is exogenously given in the partial-equilibrium textbook model. In contrast, the demand function is endogenously derived in the general equilibrium model.

We show that inefficiency arises when and only when capital endowment falls onto an intermediate range, in which circumstance both technologies are operating, but the total output is less than the social optimum. The capital-intensive technology is underutilized because by doing so, firm M can depress the rental price of capital to maximize its monopoly rent. Meanwhile, the labor-intensive technology is overused. We show that in some cases, the labor-intensive technology would be completely abandoned when both technologies are publicly available but private accessibility to the capital-intensive technology results in prolonged existence of the labor-intensive technology.

Then, we study a simple two-period dynamic model in which we compare the first-best case and the case with private capital-intensive technology. The dynamic problem is decomposed into two steps. The first step is to decide the intertemporal capital allocation (optimal saving decision). The second step is to study the intratemporal allocation across the two technologies, given the capital endowment and technology available for each period. So the results obtained from the static model are still applicable in the second step. The first-best case is fully characterized: there are six different dynamic patterns of technology adoption, depending on the initial capital endowment and the efficiency in capital goods production. Generally speaking, the capital-intensive technology is adopted earlier and utilized more when the initial capital endowment is more abundant. However, the analysis becomes much more complicated when the capital-intensive technology is initially privately accessible because firm M must consider capital allocation, market structures, and prices, both intertemporally and intratemporally, when deciding how to adopt the capital-intensive technology.

One striking result is that the new capital-intensive technology, when privately accessible, may sometimes be adopted even earlier than the case when both technologies are publicly available. This socially inefficient premature adoption of private new technology results from the fact that firm M can earn positive profits only by operating this unique technology, so it strives to implement this technology as much as it can. This departs from the standard result in the existing growth literature on technology adoption, which typically argues that adoption is inefficiently delayed due to its private accessibility, together with various diffusion frictions such as political-economy motives (Parente, 1994; Parente and Prescott, 1994, 1999); (Acemoglu and Robinson, 2001); (Comin and Hobijn, 2010; Wang, 2013). Moreover, in the aforementioned literature, the new technology is always strictly better than the old technology, independent of capital endowment, whereas it is no longer necessarily true in our model because whether the new private technology is socially more efficient than the public technology depends endogenously on relative factor abundance.

We also show that under certain conditions, there exists a nonmonotonic relation between the initial capital endowment and the equilibrium timing of adopting the capital-intensive technology. More precisely, when the initial capital endowment is larger than a certain threshold value, the technology is immediately adopted in the first period. When the initial capital endowment exceeds a higher threshold value, adoption is postponed to the second period. When the initial capital endowment exceeds an even higher threshold value, the capital-intensive technology is again immediately adopted in the first period. This nonmonotonicity result stands in contrast with the monotonic relation in the first-best case when both technologies are free and public. It is mainly because the interest rate (different from the rental price of capital) is also endogenously affected by technology adoption in the Euler equation. Given the one-period lag-free imitation, delays in the first adoption of the capital-intensive technology yield higher instantaneous profits due to capital accumulation but also cause intertemporal discounting by the interest rate. When the initial capital endowment increases from a relatively low level, firm M finds it worthwhile to wait until the next period so that capital accumulation can make the second-period instantaneous profit sufficiently larger than the instantaneous profit obtainable in the first period even after accounting for the endogenous discounting factor for profits. However, when the initial capital endowment becomes sufficiently large, the time discounting force becomes dominant, again resulting in immediate adoption.

The idea that the appropriate technology should be consistent with factor endowment can be dated back at least as far as the Heckscher–Ohlin trade model, where the mechanism is international specialization. Atkinson and Stiglitz (1969) are perhaps the first to formalize the idea that technological change is localized for a small range of capital–labor ratios, so technologies developed in rich countries are not necessarily suitable for developing countries. Basu and Weil (1998) build on that idea and study economic convergence and divergence in a Solow-type growth model. Lin (2009) explores a wide array of development issues related to the consistency of industries (technologies) with the factor endowment in developing countries. Ju et al. (2015) develop an endogenous growth model with an infinite number of technologies (industries) heterogeneous in capital intensities, in which all the factor prices are socially efficient signals to guide the first-best technology adoption and industrial upgrading. Whereas these studies assume that all technologies are publicly and freely available, the model developed here explores technology adoption when the capital-intensive technology is initially only privately accessible, and factor price signals may no longer serve as socially efficient signals due to the dynamic general equilibrium effect of the endogenous market structure.

This paper is also related to the vast literature on directed technical change, which mainly explores how the relative abundance of different factors affects the direction and the magnitude of endogenous technical change (Acemoglu (2002, 2007, 2009), Acemoglu and Zilibotti (2001), Jones (2005)). However, the analytical focus of this paper is different from that literature. Instead of exploring how relative factor abundance determines the endogenous technical change rate and favors which production factor dynamically, we mainly study the implications of the private accessibility of a given capital-intensive technology for technology adoption when capital accumulates endogenously.

In the model, firm M extracts monopoly rents from its private technology, and the industry upgrades by operating the new technology, which gradually replaces the old technology as the economy grows. These features are shared by the growing literature on vertical innovation or "creative destruction" (Aghion and Howitt, 1992, 2009; Grossman and Helpman, 1991). However, there are several key differences. First and foremost, we focus on how privately accessible and existing technology is adopted, whereas the vertical innovation literature explores how new technologies are invented. Second, the driving forces are also different. In our model, it is factor endowment and capital accumulation that determine which existing technology is superior (produces more) and how the capital-intensive technology is dynamically adopted, whereas, in the vertical innovation literature, costly R&D drives technology upgrading. Third, policy implications are different. The creative destruction literature mainly concerns how a developed economy can achieve endogenous technological progress by innovation. However, our model is mainly concerned with how to ensure that existing technology is adopted in a developing economy.[1]

The paper is structured as follows. Section 2 studies the static model, followed by the analysis of the dynamic model in Section 3. Brief policy suggestions are discussed in Section 4. The last section concludes.

2. Static model

Consider a static autarky populated by a continuum of identical households with measures equal to unity. Each household is endowed with K units of capital and L units of labor (time). Define the endowment structure as $k \equiv \frac{K}{L}$. There is only one consumption good, which can be produced with two alternative Cobb–Douglas technologies: technology 1 and technology 2. Throughout the paper, we interchangeably call them industry 1 and industry 2, respectively. The corresponding production functions are given by $F^{[1]}\left(K_1, L_1\right) = A_1 K_1^{\alpha_1} L_1^{1-\alpha_1}$ and $F^{[2]}\left(K_2, L_2\right) = A_2 K_2^{\alpha_2} L_2^{1-\alpha_2}$, where A_i, K_i, L_i and α_i are the total factor productivity, capital, labor, and capital share for technology $i \in \{1, 2\}$. Without loss of generality, assume technology 2 is more capital intensive: $0 < \alpha_1 < \alpha_2 < 1$. Following the pertinent literature, when technology 2 is adopted, it is referred to as industry upgrading. A representative household's utility function is

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}, \text{ where } \sigma \leq 1.$$

We first analyze the case when both technologies are freely and publicly available, and then we analyze the case when technology 2 is accessible only to one firm (hereafter called firm M).

2.1. Technology 2 is free and public

When both technologies are freely accessible, the market structure is endogenously perfectly competitive. The Second Welfare Theorem holds, so the competitive equilibrium can be characterized by solving the following social planner problem:

$$\begin{aligned} G(K, L) \equiv \max_{\{K_n, L_n\}_{n=1}^{2}} \; & A_1 K_1^{\alpha_1} L_1^{1-\alpha_1} + A_2 K_2^{\alpha_2} L_2^{1-\alpha_2} \\ s.t.\; & \\ & K_1 + K_2 = K, \\ & L_1 + L_2 = L, \end{aligned}$$

[1] One piece of literature focuses on information externality and studies how the risk and uncertainty affect strategic investment decisions under asymmetric information such as learning behaviors and strategic delay (Chamley, 2004). Ederington and McCalman (2009) focuses on the dynamic tradeoff that an early entry (adoption) implies a larger market share and an exogenously higher production cost. However, these are partial-equilibrium analyses, and the factor endowment plays no role.

$$K_n \geq 0, L_n \geq 0, n = 1, 2.$$

The value function $G(K, L)$ is the endogenous aggregate production function. The resource allocation problem is a standard nonlinear programming problem with a strictly concave objective function. As a result, there exists a unique solution characterized by the Kuhn–Tucker condition. Let $k_n \equiv \frac{K_n}{L_n}$ denote the capital–labor ratio used for technology n. Define

$$k_1^* \equiv \left[\left(\frac{\alpha_1}{\alpha_2}\right)^{\alpha_2}\left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{1-\alpha_2}\left(\frac{A_1}{A_2}\right)\right]^{\frac{1}{\alpha_2-\alpha_1}}, \tag{1}$$

$$k_2^* \equiv \left[\left(\frac{\alpha_1}{\alpha_2}\right)^{\alpha_1}\left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{1-\alpha_1}\left(\frac{A_1}{A_2}\right)\right]^{\frac{1}{\alpha_2-\alpha_1}}. \tag{2}$$

Observe $\frac{k_1^*}{k_2^*} = \left(\frac{\alpha_1}{\alpha_2}\right)\left(\frac{1-\alpha_2}{1-\alpha_1}\right) < 1$.

Proposition 1. *When both technologies are publicly and freely available, the market structure is perfectly competitive, and the following is true in the static equilibrium:*

(a) If $\frac{K}{L} \leq k_1^*$, only technology 1 operates and $G(K, L) = A_1 K^{\alpha_1} L^{1-\alpha_1}$.

(b) If $k_1^* < \frac{K}{L} < k_2^*$, both technologies operate with resources allocated as follows:

$$L_1 = \frac{k_2^* L - K}{k_2^* - k_1^*};\ L_2 = \frac{K - k_1^* L}{k_2^* - k_1^*}, \tag{3}$$

$$K_1 = k_1^* L_1^*;\ K_2 = k_2^* L_2^*. \tag{4}$$

where k_1^* and k_2^* are given by (1) and (2). Moreover, $G(K, L) = aK + bL$, where $a \equiv A_1\alpha_1 \left(k_1^*\right)^{\alpha_1-1}$ and $b \equiv (1-\alpha_1)A_1 \left(k_1^*\right)^{\alpha_1}$.

(c) If $\frac{K}{L} \geq k_2^*$, only technology 2 operates and $G(K, L) = A_2 K^{\alpha_2} L^{1-\alpha_2}$.

Proof. If the solution is interior, it satisfies two first-order conditions that equate the marginal productivity of labor and capital across the two different technologies, from which we obtain $k_1 = k_1^*$ and $k_2 = k_2^*$ as given by (1) and (2). Using the following factor market clearing conditions

$$k_1^* L_1 + k_2^* L_2 = K,$$
$$L_1 + L_2 = L,$$

together with (1) and (2), we obtain the equilibrium resource allocation across the two technologies (3) and (4). To satisfy the interior physical constraints $L_1 > 0$ and $L_2 > 0$, we must have

$$k_1^* < \frac{K}{L} < k_2^*.$$

Otherwise, the solution is a corner one. ■

The above proposition implies that the endogenous aggregate production function $G(K, L)$ is constant return to scale, continuously differentiable, concave, and strictly increasing in both arguments. The main results in this proposition can be intuitively illustrated in Fig. 1.

It plots the output per labor y as a function of the capital–labor ratio k. The two different technologies are represented by the two different concave curves, which cross each other at the origin and point Q, where the corresponding capital–labor ratio is denoted by

$$\widetilde{k} \equiv \left(\frac{A_1}{A_2}\right)^{\frac{1}{\alpha_2-\alpha_1}}. \tag{5}$$

Clearly, technology 1 is better than technology 2 if and only if $k \leq \widetilde{k}$. The two curves have one unique cotangent straight line $y = ak + b$ and the x-coordinates of the two tangent points M and N exactly correspond to k_1^* and k_2^* given by (1) and (2). The aggregate production function per labor ($\frac{G(K,L)}{L}$) is the convex envelope of the two technology curves. In particular, when $k_1^* < k < k_2^*$, both technologies are used simultaneously, and the aggregate production function per labor is linear (denoted by segment MN), in which case the equilibrium rental price of capital R is equal to the slope a, and the wage rate W is equal to the intercept b. When $k \leq k_1^*$, only technology 1 is operating, so $\frac{G(K,L)}{L} = A_1 k^{\alpha_1}$. When $k \geq k_2^*$, only technology 2 is operating, so $\frac{G(K,L)}{L} = A_2 k^{\alpha_2}$.

It is worth noting that capital is not subject to diminishing returns when $k_1^* < k < k_2^*$, even though there is no productivity change in either of the two specific technologies (i.e., A_1 and A_2 are fixed) nor does any nonconvexity exist. The resource reallocation during technology upgrading is what sustains the constant capital returns.

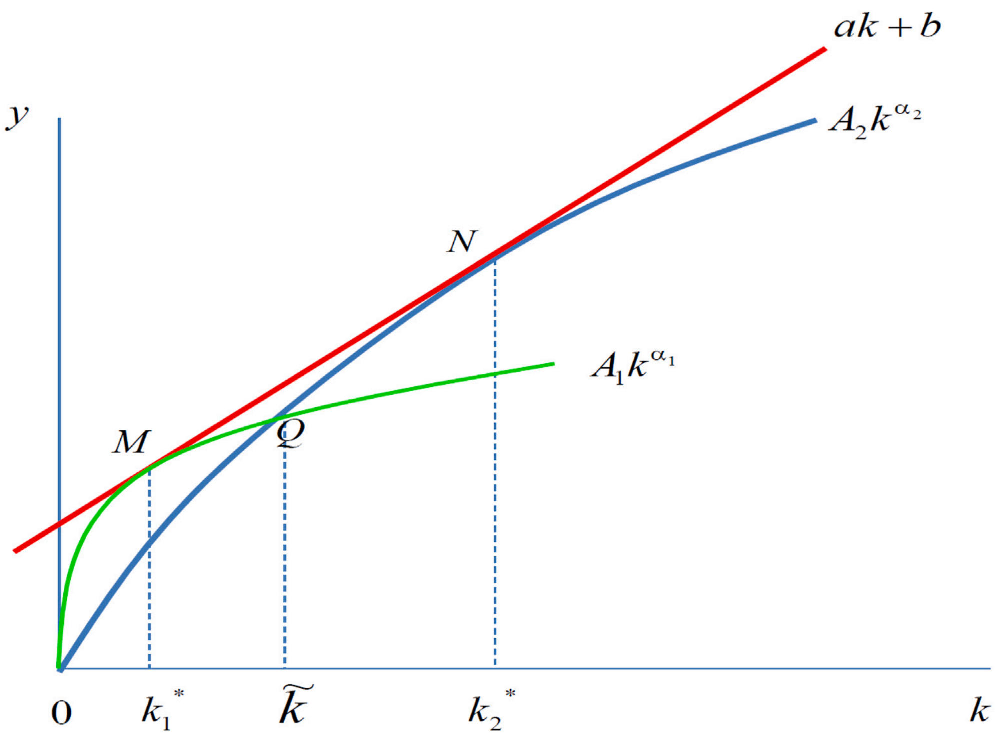

Fig. 1. Static competitive equilibrium.

2.2. Technology 2 is exclusive and private

Now suppose technology 2 is privately and exclusively accessible to firm M, which can be alternatively interpreted as the effective coalition of all potential firms that have access to technology 2.[2] Technology 1 is still publicly and freely accessible. The ownership, and hence the dividend, of each firm is equally divided among all households. We explore whether firm M operates technology 2 given that K and L. The factor markets are perfectly competitive.

Given factor prices, the unit costs for these two technologies are given, respectively, by

$$\mu_1(W,R)=\frac{R^{\alpha_1}W^{1-\alpha_1}}{A_1\alpha_1^{\alpha_1}\left(1-\alpha_1\right)^{1-\alpha_1}} \tag{6}$$

and

$$\mu_2(W,R)=\frac{R^{\alpha_2}W^{1-\alpha_2}}{A_2\alpha_2^{\alpha_2}\left(1-\alpha_2\right)^{1-\alpha_2}}. \tag{7}$$

Normalize $A_1=1$ and let $A_2=A$. Technology 2 is less costly, *i.e.*, $\mu_2(W,R)<\mu_1(W,R)$, if and only if

$$\frac{R}{W}<\psi\equiv\left[A\frac{\alpha_2^{\alpha_2}\left(1-\alpha_2\right)^{(1-\alpha_2)}}{\alpha_1^{\alpha_1}\left(1-\alpha_1\right)^{(1-\alpha_1)}}\right]^{\frac{1}{\alpha_2-\alpha_1}}. \tag{8}$$

If technology 2 is operated in equilibrium, then $\frac{R}{W}\le\psi$. The equilibrium output price P is no higher than $\mu_1(W,R)$ because technology 1 is freely available. We consider two scenarios. Scenario 1 is that firm M takes factor prices and consumers' income as exogenous. Scenario 2 is that firm M understands that its production decision would affect factor prices and aggregate income of consumers, so it takes that into account when making production and pricing decisions.

2.2.1. Scenario 1

When firm M serves the whole market demand, it solves the following:

$$\Pi=\max_{P\le\mu_1(W,R)}\left[P-\mu_2(W,R)\right]\frac{Y}{P},$$

[2] Firm M could also be imagined as a giant global company that considers making FDI in a host economy or a domestic special interest group that obtains sole permission from a foreign company to operate this new technology.

where Y is the total consumption expenditure of the economy. Firm M takes Y and factor prices W and R as exogenously given, then $P = \mu_1(W, R)$ and

$$\Pi = [1 - \frac{\mu_2(W, R)}{\mu_1(W, R)}]Y. \tag{9}$$

The total expenditure is equal to the total household wealth, or the sum of profits, labor income, and capital rental income:

$$Y = \Pi + WL + RK. \tag{10}$$

Combining (9) and (10) yields

$$Y = \frac{\mu_1(W, R)}{\mu_2(W, R)}(WL + RK).$$

In this general equilibrium environment, the market clearing conditions imply

$$AK^{\alpha_2}L^{1-\alpha_2} = \frac{WL + RK}{\mu_2(W, R)}, \tag{11}$$

where the right-hand side is the total production cost divided by unit cost, so it is equal to the total output given by the left-hand side. (7) and (11) imply

$$\frac{R}{W} = \frac{\alpha_2}{(1 - \alpha_2)\, k}, \tag{12}$$

so condition (8) is reduced to

$$k \geq k_2^*, \tag{13}$$

where k_2^* is given by (2). Recall Proposition 1 says that only technology 2 is operated if and only if $k \geq k_2^*$ when both technologies are publicly available. So the cutoff values for the capital–labor ratio are identical, independent of whether technology 2 is publicly or privately accessible. Moreover, when $k > k_2^*$, the monopoly achieves social optimality as the total output (consumption) is identical to the first-best case, departing from the standard partial-equilibrium result that monopoly is socially inefficient. This is because the negative effect of the price markup on consumption demand is exactly canceled out by the positive income effect of the extra profit on the consumption demand through the general equilibrium channel. Note that the demand function in general equilibrium is no longer exogenously given, and monopoly rents go to households and become consumers' income.

Nonetheless, when $k \geq k_2^*$, equilibrium output price and profit still differ. When technology 2 is publicly available, the final good price is equal to $\mu_2(W, R)$ given by (7). When normalized by wage, together with (13), the price in terms of labor is $\frac{\mu_2(W,R)}{W} = \frac{1}{A_2(1-\alpha_2)k^{\alpha_2}}$, and the profit is zero because of perfect competition. When technology 2 is privately accessible, the final goods price is equal to $\mu_1(W, R)$ given by (6). When normalized by wage, the price is given by $\frac{\mu_1(W,R)}{W} = \frac{\alpha_2^{\alpha_1}}{A_1\alpha_1^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}(1-\alpha_2)^{\alpha_1}k^{\alpha_1}}$ and the monopoly profit (in terms of labor) is given by

$$\frac{\Pi}{W} = \frac{\left[\left(\frac{k}{k_2^*}\right)^{\alpha_2-\alpha_1} - 1\right]}{(1 - \alpha_2)}L\,. \tag{14}$$

When $k \leq k_1^*$, the unit cost of technology 2 is strictly higher than that of technology 1, so the final good market is perfectly competitive with only technology 1 operating. When $k \in (k_1^*, k_2^*)$, firm M has no advantage in unit cost than firms using technology 1 because $\mu_2(W, R) = \mu_1(W, R)$ as $\frac{R}{W} = \psi$. As a result, the market structure is also perfectly competitive, even though technology 2 is active and privately accessible. We summarize the above findings in the following proposition.

Proposition 2. *Suppose technology 2 is exclusively and privately accessible only to firm M, and suppose firm M always takes factor prices and consumers' income as exogenous. The following is true in the static market equilibrium:*

(a) If $\frac{K}{L} \leq k_1^*$, only technology 1 operates, the market structure is perfectly competitive, and total output is $\widehat{G}(K, L) = A_1K^{\alpha_1}L^{1-\alpha_1}$.

(b) If $k_1^* < \frac{K}{L} < k_2^*$, both technologies operate, the market structure is perfectly competitive, and all quantities and prices are identical to the counterpart in Proposition 1. Moreover, the total output is $\widehat{G}(K, L) = aK + bL$, where $a \equiv A_1\alpha_1\left(k_1^*\right)^{\alpha_1-1}$ and $b \equiv (1 - \alpha_1)A_1\left(k_1^*\right)^{\alpha_1}$.

(c) If $\frac{K}{L} \geq k_2^*$, only technology 2 operates, the market structure is monopolistic, the profit of firm M is given by (14), and total output is $\widehat{G}(K, L) = A_2K^{\alpha_2}L^{1-\alpha_2}$.

Comparing Propositions 1 and 2, we conclude that factor allocation to the technologies, total output, and social welfare are independent of whether technology 2 is privately or publicly accessible in general equilibrium when firm M takes factor prices and aggregate income as exogenous.

2.2.2. Scenario 2

Firm M is now sophisticated enough to take all the general equilibrium effects into account. That is, it understands that W, R, and Y are all endogenously affected by their pricing and production decisions, so firm M takes advantage of its market power to maximize its profit. For analytical simplicity, we assume that there are two departments within firm M: a planning department and a production department. The planning department first decides the optimal amount of profit-maximizing output (Q) by considering that factor prices are endogenously affected by its output decision. Then the production department receives the output order from the planning department and takes care of the whole production process: It goes to the factor markets to hire the optimal amount of labor and rent the optimal amount of capital to minimize the total production cost for the given output target Q. However, the production department itself takes factor prices as exogenously given.[3]

To characterize the equilibrium, we use backward induction by analyzing the decision of the production department first. By Shephard Lemma, to produce Q units of output with technology 2 requires the following amount of factors:

$$L_2^*(Q,W,R) = \frac{Q}{A\left(\frac{\alpha_2}{1-\alpha_2}\right)^{\alpha_2}\left(\frac{W}{R}\right)^{\alpha_2}}; \; K_2^*(Q,W,R) = \frac{Q}{A\left(\frac{\alpha_2}{1-\alpha_2}\right)^{\alpha_2-1}\left(\frac{W}{R}\right)^{\alpha_2-1}}. \tag{15}$$

Firm M understands that it is not necessarily always optimal to grab the whole market. When technology 1 is also active, to produce Q units of output with technology 1, a competitive fringe of firms needs the following amount of inputs:

$$L_1^*(Q,W,R) = \frac{Q}{\left(\frac{\alpha_1}{1-\alpha_1}\right)^{\alpha_1}\left(\frac{W}{R}\right)^{\alpha_1}}; \; K_1^*(Q,W,R) = \frac{Q}{\left(\frac{\alpha_1}{1-\alpha_1}\right)^{\alpha_1-1}\left(\frac{W}{R}\right)^{\alpha_1-1}}.$$

Observe that

$$k_1(Q,W,R) \equiv \frac{K_1^*(Q,W,R)}{L_1^*(Q,W,R)} = \frac{\alpha_1}{1-\alpha_1}\frac{W}{R}, \tag{16}$$

$$k_2(Q,W,R) \equiv \frac{K_2^*(Q,W,R)}{L_2^*(Q,W,R)} = \frac{\alpha_2}{1-\alpha_2}\frac{W}{R}. \tag{17}$$

Combining (15), (16) and factor markets clearing conditions yields

$$\frac{K_1^*}{L_1^*} = \frac{K - \frac{Q}{A\left(\frac{\alpha_2}{1-\alpha_2}\right)^{\alpha_2-1}\left(\frac{W}{R}\right)^{\alpha_2-1}}}{L - \frac{Q}{A\left(\frac{\alpha_2}{1-\alpha_2}\right)^{\alpha_2}\left(\frac{W}{R}\right)^{\alpha_2}}} = \frac{\alpha_1}{1-\alpha_1}\frac{W}{R}, \tag{18}$$

where the second equation can be rewritten as

$$Q = \frac{A\left(\frac{\alpha_2}{1-\alpha_2}\right)^{\alpha_2}(1-\alpha_1)(1-\alpha_2)}{\alpha_2-\alpha_1}\left(\frac{R}{W}\frac{K}{L} - \frac{\alpha_1}{1-\alpha_1}\right)\left(\frac{R}{W}\right)^{-\alpha_2} L. \tag{19}$$

Observe $\left(\frac{R}{W}\right)'(Q) > 0$, because a higher output Q raises the relative demand for capital as technology 2 is more capital intensive, leading to a rise in the relative price of capital $\frac{R}{W}$. The planning department of firm M chooses output Q to maximize the profit by taking into account the impact of Q on factor prices.

Firm M's optimization problem is as follows

$$\Pi = \max_{P\le\mu_1(W,R);Q\ge 0}\left[P - \mu_2(W,R)\right]Q. \tag{20}$$

The coexistence of both technologies implies equal output price $P = \mu_1(W,R)$. Substituting it and (19) into (20), normalized by wage, yields

$$\frac{\Pi}{W} = \frac{A\left(\frac{\alpha_2}{1-\alpha_2}\right)^{\alpha_2}(1-\alpha_1)(1-\alpha_2)}{\alpha_2-\alpha_1}\left[\frac{R}{W}k - \frac{\alpha_1}{1-\alpha_1}\right]\cdot \left[\frac{\left(\frac{R}{W}\right)^{\alpha_1-\alpha_2}}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}} - \frac{1}{A(\alpha_2)^{\alpha_2}(1-\alpha_2)^{1-\alpha_2}}\right]L. \tag{21}$$

Profit maximization implies the following first-order condition about $\frac{R}{W}$ (via the choice of Q):

$$\frac{(1+\alpha_1-\alpha_2)\left(\frac{R}{W}\right)^{\alpha_1-\alpha_2}}{(\alpha_1)^{\alpha_1}}k + \frac{(\alpha_2-\alpha_1)\left(\frac{R}{W}\right)^{\alpha_1-\alpha_2-1}}{(1-\alpha_1)}\alpha_1^{1-\alpha_1} - \frac{(1-\alpha_1)^{1-\alpha_1}}{A(\alpha_2)^{\alpha_2}(1-\alpha_2)^{1-\alpha_2}}k = 0. \tag{22}$$

It can be verified that the second-order condition holds, and there exists a unique solution $\frac{R}{W}$ to (22), denoted as $\frac{R}{W} = \Gamma(k)$, where function $\Gamma(k)$ is continuously differentiable and strictly decreasing. Consequently, (21) implies that $\frac{\Pi}{W}$ is strictly increasing in k for any given L. Observe that $\Gamma(k_2^*) < \Gamma(k_1^*) = \psi$, so (22) implies that $\frac{R}{W} < \psi \Leftrightarrow k > k_1^* \Leftrightarrow Q > 0$. That is, technology 2 is operating

[3] In the appendix, we partially characterize the case when the production department of firm M no longer takes factor prices as exogenous.

if and only if $k > k_1^*$. This cutoff value is the same as in the perfectly competitive equilibrium characterized in Proposition 1. (18) and (19) jointly imply that the aggregate output is

$$\widetilde{G}(K,L) \equiv y(\frac{K}{L})L, \tag{23}$$

where

$$y(k) \equiv \left\{ \begin{array}{c} \alpha_1^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}\left[\frac{\alpha_2}{1-\alpha_2} - \Gamma(k)k\right]\Gamma^{-\alpha_1}(k) \\ +A\left(\frac{\alpha_2}{1-\alpha_2}\right)^{\alpha_2}\left(1-\alpha_1\right)\left[\Gamma(k)k - \frac{\alpha_1}{1-\alpha_1}\right]\Gamma^{-\alpha_2}(k) \end{array} \right\} \frac{\left(1-\alpha_2\right)}{\left(\alpha_2-\alpha_1\right)}. \tag{24}$$

Firm M serves only a fraction of the market, so

$$Q(\frac{R}{W}) < AK^{\alpha_2}L^{1-\alpha_2}. \tag{25}$$

By revoking (17), (25) holds if and only if $\frac{R}{W} < \frac{\alpha_2}{(1-\alpha_2)k_2^*}$. Combining (22), it implies that

$$k < k^* \equiv \left[\frac{\alpha_2\left(1-\alpha_1\right)}{\alpha_1\alpha_2 - \alpha_1^2 + \alpha_2 - \alpha_2^2}\right]^{\frac{1}{\alpha_2-\alpha_1}} k_2^*. \tag{26}$$

Let Q_i^m and Q_i^c denote the output produced with technology $i \in \{1,2\}$ when technology 2 is private (monopoly) and public (competitive market), respectively.

Lemma 3. *In Scenario 2, when $k \in (k_1^*, k^*)$, both technologies are operating. Moreover, $Q_1^m > Q_1^c$, $Q_2^m < Q_2^c$, $Q_1^m + Q_2^m < Q_1^c + Q_2^c$.*

Proof. Please see Appendix B. ■

The intuition is that the profit of firm M decreases with the relative capital price, so firm M indirectly depresses this price by underutilizing the capital-intensive technology. Accordingly, the labor-intensive technology is overly used compared with the first-best allocation. The market equilibrium is socially inefficient.

When $k = k^*$, only technology 2 is operating and

$$\frac{R}{W} = \left[\frac{\alpha_2^{1-\alpha_2}\left(1-\alpha_1\right)^{2-\alpha_1}\alpha_1^{\alpha_1}}{A\left(1-\alpha_2\right)^{1-\alpha_2}\left[\alpha_1\alpha_2 - \alpha_1^2 + \alpha_2 - \alpha_2^2\right]}\right]^{\frac{1}{\alpha_1-\alpha_2}}.$$

The profit (21) is given by

$$\Pi = \frac{\left(\alpha_2-\alpha_1\right)^2 WL}{\left(1-\alpha_2\right)\left(\alpha_1\alpha_2 - \alpha_1^2 + \alpha_2 - \alpha_2^2\right)}. \tag{27}$$

Whenever $k \geq k^*$, only technology 2 operates, and the market structure is a monopoly. Since the Inada condition is satisfied, the aggregate output must be $AK^{\alpha_2}L^{1-\alpha_2}$ even though firm M can choose to sell only part of the output. Substituting $k = k^*$ into (14) also yields (27).

Proposition 4. *Suppose technology 2 is exclusively and privately accessible only to firm M. Suppose the planning department of firm M chooses both the quantity and the price of output to maximize firm profit by internalizing the impact of its decision on factor prices, while the production department minimizes the production cost by taking the output target (set by the planning department) and factor prices as given. Then the following is true in the static market equilibrium:*

(1) When $k \leq k_1^*$, only technology 1 is adopted, $\frac{R}{W} = \frac{\alpha_1}{(1-\alpha_1)k}$, and the total output is $\widetilde{G}(K,L) = K^{\alpha_1}L^{1-\alpha_1}$;

(2) When $k \in (k_1^*, k^*)$, both technologies are operating, $\frac{R}{W}$ is uniquely determined by (22), the profit is given by (21), and the total output is given by (23);

(3) When $k \geq k^*$, only technology 2 is operating with $\frac{R}{W} = \frac{\alpha_2}{(1-\alpha_2)k}$, profit given by (14), and output $\widetilde{G}(K,L) = AK^{\alpha_2}L^{1-\alpha_2}$.

We can see that the equilibrium is socially efficient when $k \leq k_1^*$ as it is perfectly competitive with only technology 1. The capital-intensive technology is adopted only when the capital–labor ratio exceeds the same threshold value for capital, irrespective of the public accessibility to technology 2, but the total output is smaller than the first best when $k \in (k_1^*, k^*)$, and technology 1 is "overly" used in the sense that industry 1 would have been abandoned if technology 2 were publicly available when $k \in (k_2^*, k^*)$, but it still operates when technology 2 is monopolized. Social efficiency is restored when $k \geq k^*$ even though the market structure is a monopoly and the prices are different from those in perfect competition (it also differs from the artificial social planner problem). This is because the negative price effect due to monopoly markup exactly cancels out the positive income effect due to the general equilibrium nature that monopoly rent of firm M becomes part of the households' income. To summarize, a nonmonotonic relationship exists between the capital–labor ratio and the social efficiency of technology adoption.

2.2.3. Further discussions

We now briefly discuss what happens when some assumptions deviate from the previous model setting.

One deviation is to introduce an explicit adoption cost of technology 2. Suppose technology 2 is still privately accessible, but now it requires some fixed cost to operate it. Denote the entry cost by η (in terms of capital). Technology 2 is adopted if and only if $\Pi(K,L) \geq R\eta$. Consider the case when k is sufficiently large that only technology 2 is operating if and only if

$$\frac{\left[\left(\frac{K-\eta}{Lk_2^*}\right)^{\alpha_2-\alpha_1}-1\right]}{\left(1-\alpha_2\right)}L \geq R\eta,$$

where $\frac{R}{W} = \frac{\alpha_2 L}{(1-\alpha_2)(K-\eta)}$, so

$$\left(\frac{K-\eta}{Lk_2^*}\right)^{\alpha_2-\alpha_1} - 1 \geq \frac{\alpha_2}{(K-\eta)}\eta.$$

There exists a unique $K^*(L,\eta)$ such that the above inequality holds if and only if $K \geq K^*(L,\eta)$, where $K_1^*(L,\eta) > 0$ and $K_2^*(L,\eta) > 0$. Obviously, $K^*(L,\eta) > k^* \cdot L$. The total output is $A\,(K-\eta)^{\alpha_2}\, L^{1-\alpha_2}$. When technology 2 is not adopted, the total output is $K^{\alpha_1} L^{1-\alpha_1}$. So when

$$A\,(K-\eta)^{\alpha_2}\, L^{1-\alpha_2} > K^{\alpha_1} L^{1-\alpha_1}, \tag{28}$$

or alternatively, when K is sufficiently large, adopting technology 2 is socially optimal despite the monopoly market structure. The economic intuition is the same as before. However, if adoption cost η is so large that (28) is not satisfied, a monopoly with only technology 2 is no longer socially optimal.

The second deviation is to assume that there exist multiple firms that have access to technology 2. In other words, there is more than one firm M. Suppose there are n symmetric firms with $n \geq 2$. If these firms are engaged in Bertrand-type competition, then the equilibrium outcome is equivalent to that technology 2 is public, returning to the first-best scenario. If, on the other extreme, all these firms can form a coalition with full commitment, then the equilibrium outcome is the same as the monopoly case analyzed above.

If these firms are engaged in Cournot-type interaction in the oligopoly, we must analyze their strategic decisions in the general equilibrium framework. To further simplify the analysis, imagine $n = 2$, so there are two symmetric firms, M and M'. Suppose these two firms take factor prices and households' total income both as exogenous (same as Scenario 1). Firm M solves the following optimization problem:

$$\Pi_M = \max_{P \leq \mu_1(W,R)} \left[P - \mu_2(W,R)\right] \left(\frac{Y}{P} - Q_{M'}\right),$$

where $Q_{M'}$ is the output level produced by firm M'. Firm M takes $Q_{M'}$ as exogenous. The first-order condition with respect to P implies

$$P = \min\{\mu_1(W,R), \sqrt{\frac{\mu_2(W,R)Y}{Q_{M'}}}\}.$$

(i) When $\sqrt{\frac{\mu_2(W,R)Y}{Q_{M'}}} < \mu_1(W,R)$, we obtain

$$P = \sqrt{\frac{\mu_2(W,R)Y}{Q_{M'}}}.$$

By symmetry and factor markets clearing conditions, we have

$$Q_{M'} = Q_M = A\left(\frac{K}{2}\right)^{\alpha_2}\left(\frac{L}{2}\right)^{1-\alpha_2}.$$

Final good market clearing condition

$$Y = 2PQ_M = 2\sqrt{Q_{M'}\mu_2(W,R)Y}.$$

The above three equations jointly imply

$$\begin{aligned} P &= 2\mu_2(W,R), \\ \Pi_M &= \mu_2(W,R)A\left(\frac{K}{2}\right)^{\alpha_2}\left(\frac{L}{2}\right)^{1-\alpha_2}, \\ Y &= 2\mu_2(W,R)AK^{\alpha_2}L^{1-\alpha_2}, \end{aligned}$$

where $\frac{R}{W} = \frac{\alpha_2}{1-\alpha_2}\frac{L}{K}$. To ensure $\sqrt{\frac{\mu_2(W,R)Y}{Q_{M'}}} < \mu_1(W,R)$, we must have $2\mu_2(W,R) < \mu_1(W,R)$, or equivalently,

$$\frac{R}{W} < 2^{-\frac{1}{\alpha_2-\alpha_1}}\psi, \tag{29}$$

where ψ is given by (8). (29) is equivalent to

$$\frac{K}{L} > 2^{\frac{1}{\alpha_2-\alpha_1}} k_2^*,$$

where k_2^* is given by (2).

(ii) When $\sqrt{\frac{\mu_2(W,R)Y}{Q_{M'}}} \geq \mu_1(W,R)$, or equivalently, $2\mu_2(W,R) \geq \mu_1(W,R)$, then by using a similar method, we obtain

$$\begin{aligned} P &= \mu_1(W,R), \\ \Pi_M &= \frac{\mu_1(W,R)-\mu_2(W,R)}{\mu_2(W,R)}(RK+WL), \\ Q_{M'} &= Q_M = A\left(\frac{K}{2}\right)^{\alpha_2}\left(\frac{L}{2}\right)^{1-\alpha_2}, \\ Y &= \mu_1(W,R)AK^{\alpha_2}L^{1-\alpha_2}, \end{aligned}$$

where $\frac{R}{W} = \frac{\alpha_2}{1-\alpha_2}\frac{L}{K}$. We require

$$\psi \geq \frac{R}{W} \geq 2^{-\frac{1}{\alpha_2-\alpha_1}}\psi$$

or equivalently

$$k_2^* \leq \frac{K}{L} \leq 2^{\frac{1}{\alpha_2-\alpha_1}} k_2^*,$$

which the first inequality ensures that $\Pi_M \geq 0$, and the second one ensures that the limit pricing constraint is binding.

It is easy to show that when $\frac{K}{L} < k_2^*$, the market equilibrium is identical to that in Scenario 1. These analyses can be easily generalized to the case with firm number $n \geq 3$. To summarize, when multiple firms can access the new technology, the market equilibrium is still always efficient even when the market structure is oligopolistic, and these firms are engaged in Cournot competition. However, if the firms internalize their output impact on factor prices when making output decisions, inefficiency arises, and the capital-intensive technology is underutilized when the capital–labor ratio falls onto some intermediate range because the logic in Scenario 2 still follows, as long as these firms are not engaged in Bertrand competition.

3. Dynamic model

Now we study the dynamic pattern of technology adoption associated with possible changes in the market structure. Again, we first characterize the benchmark case, namely the socially efficient equilibrium where both technologies are publicly and freely available. Then we explore what happens when the capital-intensive technology is initially privately accessible, where we stay with the same assumption as in Scenario 2 in the static model. That is, firm M takes advantage of its market power on factor prices. A comparison of the two cases is discussed afterward. The key insights can be illustrated with a simple two-period model.[4]

3.1. Technology 2 is free and public

A representative household is endowed with K_0 capital and L labor. E_t and C_t denote, respectively, the capital used for production and consumption at period $t \in \{1,2\}$. A capital good is produced with AK technology and cannot be used for consumption. All the capital used for production fully depreciates. Consumption goods are perishable. The market structure is endogenously perfectly competitive in the two periods. The second welfare theorem applies, and the artificial social planner solves the following problem:

$$\max_{C_1,C_2,E_1,E_2} \frac{C_1^{1-\sigma}-1}{1-\sigma} + \beta\frac{C_2^{1-\sigma}-1}{1-\sigma}, \text{ where } \sigma \in (0,1]$$

subject to

$$\begin{aligned} E_2 &= \xi(K_0 - E_1), \\ C_1 &\leq G\left(E_1, L\right), \\ C_2 &\leq G\left(E_2, L\right), \\ E_1 &\geq 0, E_2 \geq 0, C_1 \geq 0, C_2 \geq 0, \\ &K_0 \text{ and } L \text{ are given,} \end{aligned}$$

where ξ is a parameter capturing the investment-specific technological progress (hereafter, ISTP *a la* Greenwood et al. (1997)) in the capital good sector, and $G(\cdot, L)$ is given by Proposition 1. There are nine possible patterns of technology adoption. We assume $\beta\xi > 1$ so that ISTP is sufficiently quick to rule out technology downgrading. The remaining six patterns are listed in Table 1:

For example, Pattern 4 refers to that only technology 1 is used in period 1 and both technologies are used in period 2.

[4] For infinite-period models with infinite technologies (industries), please refer to Ju et al. (2015).

Table 1
First-best patterns.

Pattern	1	2	3	4	5	6
Period 1	1	2	1, 2	1	1	1, 2
Period 2	1	2	1, 2	1, 2	2	2

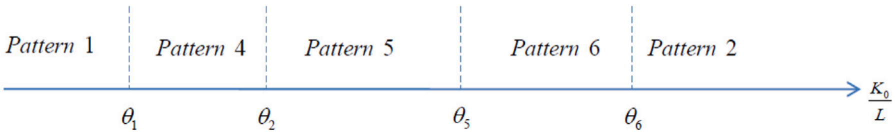

Fig. 2. Dynamic pattern of technology adoption when technology 2 is free and $\beta\xi \geq \left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{\sigma}$.

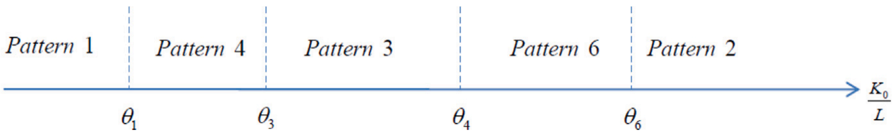

Fig. 3. Dynamic pattern of technology adoption when technology 2 is free and $\beta\xi < \left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{\sigma}$.

Proposition 5. *Suppose both technologies are public and free, $\beta\xi > 1$ and $\sigma \in (0, 1]$. When $\beta\xi \geq \left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{\sigma}$, the dynamic equilibrium follows Pattern 1 when $\frac{K_0}{L} \in (0, \theta_1]$, Pattern 4 when $\frac{K_0}{L} \in (\theta_1, \theta_2)$, Pattern 5 when $\frac{K_0}{L} \in [\theta_2, \theta_5]$, Pattern 6 when $\frac{K_0}{L} \in (\theta_5, \theta_6)$, and Pattern 2 when $\frac{K_0}{L} \in [\theta_6, \infty)$. When $\beta\xi < \left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{\sigma}$, the dynamic equilibrium follows Pattern 1 when $\frac{K_0}{L} \in (0, \theta_1]$, Pattern 4 when $\frac{K_0}{L} \in (\theta_1, \theta_3]$, Pattern 3 when $\frac{K_0}{L} \in (\theta_3, \theta_4)$, Pattern 6 when $\frac{K_0}{L} \in [\theta_4, \theta_6)$, and Pattern 2 when $\frac{K_0}{L} \in [\theta_6, \infty)$. The threshold values are given by*

$$
\begin{aligned}
\theta_1 &\equiv \frac{1 + \xi^{-1} (\beta\xi)^{\frac{1}{-\alpha_1+1+\alpha_1\sigma}}}{(\beta\xi)^{\frac{1}{-\alpha_1+1+\alpha_1\sigma}}} k_1^*, \\
\theta_2 &\equiv \left[(\beta\xi)^{\frac{1}{-\sigma\alpha_1+\alpha_1-1}} \left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{\frac{\sigma}{\sigma\alpha_1-\alpha_1+1}} + \frac{\alpha_2 (1-\alpha_1)}{\alpha_1 (1-\alpha_2) \xi} \right] k_1^*, \\
\theta_3 &\equiv \frac{(\beta\xi)^{\frac{1}{\sigma}} + \xi\alpha_1 - (1-\alpha_1)}{\alpha_1 \xi} k_1^*, \\
\theta_4 &\equiv \left[\frac{1}{\xi} + \frac{(\beta\xi)^{-\frac{1}{\sigma}} - (1-\alpha_2)}{\alpha_2} \right] \frac{\alpha_2 (1-\alpha_1)}{\alpha_1 (1-\alpha_2)} k_1^*, \\
\theta_5 &\equiv \left[\frac{\alpha_2}{\alpha_1} \left[\beta\xi^{(1-\sigma)\alpha_2} \left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{(1-\alpha_2)((1-\sigma))} \right]^{\frac{1}{1-\alpha_2+\sigma\alpha_2}} + 1 \right] k_1^*, \\
\theta_6 &\equiv \frac{\alpha_2 (1-\alpha_1)}{\alpha_1 (1-\alpha_2)} \left[1 + \xi^{-1} (\beta\xi)^{\frac{1}{-\alpha_2+1+\alpha_2\sigma}} \right] k_1^*.
\end{aligned}
\tag{30}
$$

Proof. Please see Appendix C. ■

As shown in the proof, we first explore the equilibrium choice of technologies in each period for a given intertemporal capital allocation, which is the static problem characterized in Proposition 1. Then we make sure that the intertemporal output satisfies the Euler equation so that the dynamic saving decision is optimal. The six threshold values in the above proposition are found by ensuring that the domain of capital allocation for each period is consistent with the corresponding technology choice in that period.

This proposition states that the dynamic pattern depends on the initial capital–labor ratio and the ISTP parameter ξ. The results can be intuitively illustrated by the following two figures (see Fig. 3):

To summarize, there always exists a unique equilibrium in which a higher initial capital–labor ratio implies weakly quicker adoption of the capital-intensive technology. Moreover, Pattern 3 and Pattern 5 are mutually exclusive and the former (both technologies are used in both periods) occurs only if the ISTP is sufficiently slow ($\beta\xi < \left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{\sigma}$).

Table 2
Dynamic patterns when technology 2 is private.

Patterns	AA	AB	AC	BA	BB	BC	CA	CB	CC
Period 1	1, 2	1, 2	1, 2	2	2	2	1	1	1
Period 2	1, 2	2	1	1, 2	2	1	1, 2	2	1

3.2. Technology 2 is initially private and exclusive

Now consider a dynamic environment identical to the previous one except that technology 2 is privately accessible to only firm M, which decides whether and when to implement this new technology. Technology 1 is still public and free. If technology 2 is implemented in period 1, then this technology becomes publicly known in period 2 because people can successfully imitate it after one period of operation. If adoption of technology 2 is delayed until period 2, the monopoly rent is reaped in period 2. The ownership shares of all the firms are equally divided among all the households. Since the second welfare theorem is not applicable, we have to solve the decentralized optimal decisions of households and firms. A representative household solves

$$\max \frac{C_1^{1-\sigma}-1}{1-\sigma}+\beta\frac{C_2^{1-\sigma}-1}{1-\sigma}$$

subject to

$$\begin{aligned} P_1C_1+\frac{P_2C_2}{\widetilde{R}} &\leq \left(W_1L+R_1E_1+\Pi_1\right)+\frac{\left(W_2L+R_2E_2+\Pi_2\right)}{\widetilde{R}}, \\ E_2 &= \xi(K_0-E_1), \\ E_1 &\geq 0, E_2\geq 0, C_1\geq 0, C_2\geq 0, \\ &K_0 \text{ is given.} \end{aligned} \tag{31}$$

Household optimization yields

$$\beta\left(\frac{C_2}{C_1}\right)^{-\sigma}=\frac{P_2}{P_1\widetilde{R}}, \tag{32}$$

which, together with (31), implies

$$C_1=\frac{\left(W_1L+R_1E_1+\Pi_1\right)+\frac{(W_2L+R_2E_2+\Pi_2)}{\widetilde{R}}}{P_1+\beta P_1\left(\frac{P_2}{\beta P_1\widetilde{R}}\right)^{1-\frac{1}{\sigma}}}, \tag{33}$$

$$C_2=C_1\left(\frac{P_2}{\beta P_1\widetilde{R}}\right)^{-\frac{1}{\sigma}}. \tag{34}$$

To ensure positive consumption in both periods, we must have the intertemporal interest rate

$$\widetilde{R}=\frac{\xi R_2}{R_1}. \tag{35}$$

Substituting (35) back to (32) yields

$$\beta\xi\left(\frac{C_2}{C_1}\right)^{-\sigma}=\frac{P_2/R_2}{P_1/R_1}. \tag{36}$$

All firms maximize their total profit. Those with access to only the public technology take factor prices as given. However, firm M understands that it can potentially affect the relative factor prices via its output decision through the general equilibrium channel when technology 2 is operated for the first time. Firm M has three options. Option 1 is to start operating technology 2 in period 1($\Pi_1>0;\Pi_2=0$); Option 2 is to first start operating technology 2 in period 2 ($\Pi_1=0;\Pi_2>0$); Option 3 is never to operate technology 2 ($\Pi_1=\Pi_2=0$). There are nine possible patterns of technology adoption, as summarized in Table 2. Pattern XY refers to that technology choice in period 1 is X∈ {A,B,C} and technology choice in period 2 is Y∈ {A,B,C}, where A means both technologies are adopted in that period, B means only technology 2 is adopted in that period, and C means only technology 1 is adopted in that period. Compared with Table 1, there are three more patterns (*i.e.*, AC, BC, BA) in Table 2 as we have to explore whether these industrial downgrading cases could be ruled out or not.

As before, the dynamic problem is analyzed in two steps. First, dynamic capital allocation (E_1 and E_2) is determined in the optimal saving decision. Second, given E_1 and E_2, the optimal technology adoption decision is made, which affects the market structure in both periods. For Option 1, the second-period problem is identical to the perfect competition case in Section 2.1 because both technologies become publicly available in period 2; the first-period problem is identical to that in Section 2.2. For Option 2, the second-period problem is identical to that in Section 2.2, whereas the first-period problem is to solve a perfect competition model with only technology 1. For Option 3, the optimization in both periods must be consistent with the static analysis in Section 2.2 to justify why technology 2 is never adopted.

The necessary and sufficient conditions for the existence of each of the nine different patterns have to be found first. Then we can determine the optimal technology adoption choice of firm M based on the present value of profit from each possible adoption strategy for given initial conditions. Technology 2 is first adopted in period 1 rather than period 2 if and only if $\Pi_1 > \frac{\Pi_2}{\widetilde{R}}$, where $\widetilde{R}$ is endogenously determined in the general equilibrium.

Lemma 6. *Suppose $\beta\xi > 1$. Patterns AC, BA, and BC never occur.*

Proof. Refer to Appendix D.

The intuition is that the consumption in the second period should be no smaller than the first period, as implied by the Euler equation and $\beta\xi > 1$. Patterns AC, BC, and BA all imply the opposite, so they cannot occur in equilibrium. Therefore, we have six different patterns left, the same as in Table 1 in Section 3.1, but the market structure may not always be perfect competition depending on when technology 2 is first adopted.

Proposition 7. *Suppose $\sigma < 1$ and $\beta\xi > 1$. There exists a nonempty interval $(\theta_0, \theta_1]$ such that for any $\frac{K_0}{L} \in (\theta_0, \theta_1]$, technology 2 is adopted in period 2 when it is privately accessible (Pattern CA), but technology 2 is never adopted when it is publicly available. Here θ_1 is given by (30) and θ_0 is uniquely determined by the following equation*

$$\beta\xi\left(\frac{y(\xi(\frac{K_0}{L}-\theta_0))}{\theta_0^{\alpha_1}}\right)^{-\sigma} = \left(\frac{1}{\theta_0 \Gamma(\xi(\frac{K_0}{L}-\theta_0))}\frac{\alpha_1}{1-\alpha_1}\right)^{1-\alpha_1}, \tag{37}$$

where $y(\cdot)$ is defined in (24) and $\Gamma(\cdot)$ is defined in (22).

Proof. See Appendix E. ■

This proposition indicates that the capital-intensive technology, when privately accessible, may be prematurely implemented from the social efficiency point of view. It occurs for two reasons. First, firm M would earn a positive profit whenever the market can support the adoption, but no adoption means zero profit. Therefore, as a monopolist of this capital-intensive technology, the profit-seeking firm M is always incentivized to implement technology 2 unless it is not supportable by the market. Second, the initial capital endowment has to be sufficiently large so that savings can make capital in the second period abundant enough to support the capital-intensive technology. The increase in saving is feasible in the dynamic equilibrium because consumptions in the two periods are sufficiently substitutable (intertemporal elasticity of substitution of consumption $\frac{1}{\sigma}$ is larger than unity).[5]

Lemma 8. *Suppose $\sigma = 1$ and $\beta\xi > \frac{1-\alpha_1}{1-\alpha_2}$. The market equilibrium is Pattern CC when $\frac{K_0}{L} \in (0, \widetilde{\theta}_1]$ and it is Pattern CA when $\frac{K_0}{L} \in (\widetilde{\theta}_1, \widetilde{\theta}_2)$. Pattern CB is supportable by the market when $\frac{K_0}{L} \in [\widetilde{\theta}_2, \infty)$; Pattern AB is supportable by the market when $\frac{K_0}{L} \in (\widetilde{\theta}_0, \widetilde{\theta}_5)$; Pattern BB is supportable by the market when $\frac{K_0}{L} \in [\widetilde{\theta}_5, \infty)$, where*

$$\begin{aligned}
\widetilde{\theta}_0 &\equiv \left(\beta\frac{\alpha_2}{\alpha_1}+1\right)k_1^*\\
\widetilde{\theta}_1 &\equiv \frac{1+\beta}{\beta\xi}k_1^*;\\
\widetilde{\theta}_2 &\equiv \frac{\alpha_1\left(1-\alpha_1\right)}{\alpha_1\alpha_2-\alpha_1^2+\alpha_2-\alpha_2^2}\frac{k^*}{\beta\xi}+\frac{k^*}{\xi};\\
\widetilde{\theta}_5 &\equiv \beta k^*+k^*.
\end{aligned}$$

Proof. Refer to Appendix F. ■

The above lemma can be illustrated by Fig. 4.

First of all, "a pattern being supportable by the market" simply means that the pattern is feasible to implement as a market equilibrium to the extent that firm M has not yet decided whether it is the most profitable choice, so it is a necessary but insufficient condition for the pattern to be a market equilibrium. When there are multiple patterns supportable by the market over a certain interval, firm M chooses the most profitable pattern for itself, which is also the market equilibrium. If there is a unique pattern supportable by the market over some interval, then this pattern must be the market equilibrium for this interval. In addition, multiple equilibria exist if two patterns deliver the same highest total profit.

Notice that Pattern AA is not supportable by the market as it is ruled out by condition $\beta\xi > \frac{1-\alpha_1}{1-\alpha_2}$. This figure indicates that there is a unique equilibrium pattern when $\frac{K_0}{L} < \widetilde{\theta}_0$, qualitatively similar to Fig. 2 in terms of intertemporal technology choices and their ordering along the dimension of initial capital–labor ratio. Observe that $\widetilde{\theta}_1 = \theta_1$ when $\sigma = 1$. It means that the threshold value of the initial capital–labor ratio for the capital-intensive technology to be adopted is independent of whether this technology is publicly or

[5] It is shown in the proof that the result in this proposition no longer holds when $\sigma = 1$.

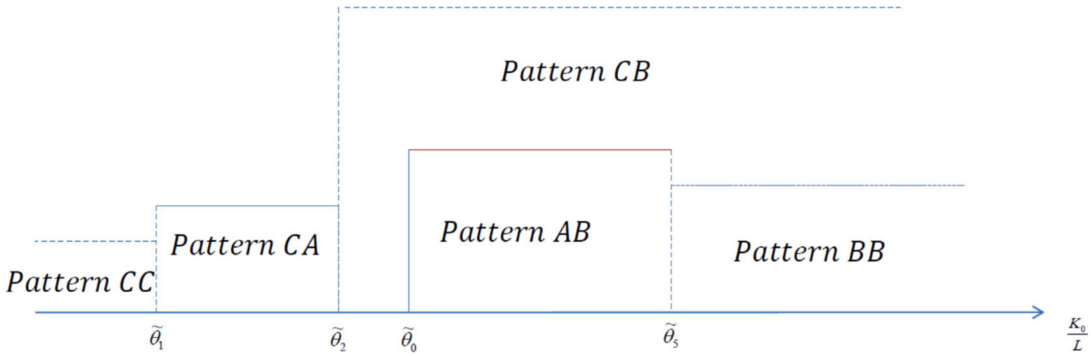

Fig. 4. Market-supportable patterns of dynamic technology adoption when technology 2 is privately accessible.

privately accessible in this dynamic environment, preserving the same property in the static economy (recall Proposition 4). When $\frac{K_0}{L} \in [\widetilde{\theta}_0, \widetilde{\theta}_5]$, it is obvious that Pattern AB is the equilibrium ($\frac{\Pi_{CB}}{\widetilde{R}} < \Pi_{AB}$) when β is sufficiently small. When β is large enough, the equilibrium could be Pattern CB, depending on the specific values for different parameters.[6]

Note that $\widetilde{\theta}_2 < \theta_2 < \widetilde{\theta}_0$ when $\sigma = 1$ and $\beta\xi > \frac{1-\alpha_1}{1-\alpha_2}$. It means that when $\frac{K_0}{L} \in (\widetilde{\theta}_2, \theta_2)$, the labor-intensive technology is completely abandoned in period 2 if the capital-intensive technology is private, whereas the labor-intensive technology is still operating (together with the capital-intensive technology) in period 2 when the capital-intensive technology is public. In other words, private access to the capital-intensive technology leads to premature abandonment of the labor-intensive technology and overutilization of the capital-intensive technology from a social efficiency point of view. The reason is that firm M wants to extract as much profit as possible by utilizing this private technology, similar to the case in Proposition 7. These results are in stark contrast with the static model, which predicts that private accessibility could result in overutilization of the old technology (Proposition 4 in Section 2.2).

Proposition 4 shows that when both technologies are publicly available, the capital-intensive technology is adopted weakly earlier whenever the initial capital endowment becomes larger. Is such a monotonic relationship still valid when the capital-intensive technology is private? The following proposition says no.

For the convenience of exposition, define

$$x_{1,2} \equiv \frac{1}{2}\left(\frac{\beta\frac{\alpha_2}{\alpha_1}+1}{A^{\frac{1}{1-\alpha_1}}}\right)^{\frac{\alpha_2-\alpha_1}{1-\alpha_1}}\left((1+\beta) \mp \sqrt{(1+\beta)^2 - 4\beta\left[\frac{(1+\beta)}{\beta\xi\left(\beta\frac{\alpha_2}{\alpha_1}+1\right)}\right]^{\frac{\alpha_2-\alpha_1}{1-\alpha_1}}}\right),$$

$$\upsilon_{1,2} \equiv \frac{\left(\beta\frac{\alpha_2}{\alpha_1}+1\right)^{\alpha_2-\alpha_1}\left[\left(\frac{\alpha_1}{\alpha_2}\right)^{\alpha_1}\left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{1-\alpha_1}\frac{\alpha_2(1-\alpha_1)}{\alpha_1\alpha_2-\alpha_1^2+\alpha_2-\alpha_2^2}\right]^{\frac{1-\alpha_1}{1-\alpha_2}}}{\left[\frac{1}{2}\left((1+\beta)\mp\sqrt{(1+\beta)^2-4\beta\left(\frac{1+\beta}{\beta\xi\left(\beta\frac{\alpha_2}{\alpha_1}+1\right)}\right)^{\frac{\alpha_2-\alpha_1}{1-\alpha_1}}}\right)\right]^{\frac{(1-\alpha_1)(\alpha_2-\alpha_1)}{1-\alpha_2}}}.$$

Proposition 9. *Suppose $\sigma = 1$, $\beta\xi \geq 1$ and $\frac{K_0}{L} \in [\left(\beta\frac{\alpha_2}{\alpha_1}+1\right)k^*, \infty)$. The dynamic market equilibrium is the following: [1] When $A > \upsilon_1$, Pattern BB is realized if $\frac{K_0}{L} \in [\left(\beta\frac{\alpha_2}{\alpha_1}+1\right)k^*, x_1) \cup (x_2, \infty)$ and Pattern CB is realized if $\frac{K_0}{L} \in (x_1, x_2)$. Both patterns are equilibria when $\frac{K_0}{L} = x_1$ or x_2. [2] When $A \in (\upsilon_2, \upsilon_1)$, Pattern BB is realized if $\frac{K_0}{L} \in (x_2, \infty)$ and Pattern CB is realized if $\frac{K_0}{L} \in [\left(\beta\frac{\alpha_2}{\alpha_1}+1\right)k^*, x_2)$. Both patterns are equilibria when $\frac{K_0}{L} = x_2$; [3] When $A < \upsilon_2$, Pattern BB is realized whenever $\frac{K_0}{L} \in [\left(\beta\frac{\alpha_2}{\alpha_1}+1\right)k^*, \infty)$.*

Proof. See Appendix F. ■

Part [1] of this proposition is most interesting. It says that there exists a nonmonotonic relationship between the optimal time to adopt the capital-intensive technology and the initial capital–labor ratio $\frac{K_0}{L}$. More specifically, when $\frac{K_0}{L}$ increases across the

[6] For example, $\Pi_{AB} > \frac{\Pi_{CB}}{\widetilde{R}}$ when $\alpha_1 = \frac{1}{3}$ and $\alpha_2 = \frac{2}{3}$, or when $\alpha_1 = \frac{1}{2}$ and $\alpha_2 = 1$, or when $\alpha_1 = 0$ and $\alpha_2 = \frac{1}{2}$, but the opposite is true when $\alpha_1 = \frac{1}{4}$ and $\alpha_2 = \frac{1}{2}$.

threshold x_1, surprisingly, the adoption of the capital-intensive technology is delayed from the first period to the second period. To understand why, observe that, on the one hand, the current value of first-period profit Π_{BB} increases with $\frac{K_0}{L}$ in Pattern BB:

$$\Pi_{BB} = \left[\left(A \left[\frac{K_0/L}{\beta \frac{\alpha_2}{\alpha_1} + 1} \right]^{\alpha_2 - \alpha_1} \right)^{\frac{1}{1-\alpha_1}} - 1 \right] W_1 L.$$

On the other hand, the discounted present value of the second-period profit also increases with $\frac{K_0}{L}$ in Pattern CB:

$$\frac{\Pi_{CB}}{\widetilde{R}_{CB}} = \left(1 - \left[A \left(\frac{K_0 \beta \xi}{L(1+\beta)} \right)^{\alpha_2 - \alpha_1} \right]^{-\frac{1}{1-\alpha_1}} \right) \beta L W_1.$$

Mathematically, it turns out that $\frac{\Pi_{CB}}{\widetilde{R}_{CB}}$ exceeds Π_{BB} when $\frac{K_0}{L} \in (x_1, x_2)$. However, when $\frac{K_0}{L} > x_2$, $\widetilde{R}_{CB}$ increases with $\frac{K_0}{L}$ sufficiently more than Π_{CB}, so it pays to switch back to Pattern BB.

The economic intuition is as follows. The private accessibility to this new technology enables firm M to extract more rents by deliberately postponing the implementation till capital becomes more abundant. An increase in initial capital endowment makes second-period capital larger due to the consumption-smoothing motives of consumers, but it also increases the intertemporal interest rate $\widetilde{R}_{CB}$ to discourage adoption delay.[7] It turns out that the waiting benefit is smaller than the waiting cost when $\frac{K_0}{L} \in [\left(\beta \frac{\alpha_2}{\alpha_1} + 1\right) k^*, x_1)$, while the opposite is true when the initial capital–labor ratio becomes larger: $\frac{K_0}{L} \in (x_1, x_2)$, and then the benefit exceeds the cost again when the capital becomes even more abundant: $\frac{K_0}{L} > x_2$. An immediate policy implication is that a limited amount of foreign aid (by increasing $\frac{K_0}{L}$) may sometimes result in a delay in the adoption of capital-intensive technology, different from the first-best case characterized by Proposition 4.

Another distinctive feature of this dynamic model is that there may exist multiple equilibria. This indeterminacy of technology adoption and industry upgrading results from the indirect price manipulations by firm M through its adoption and quantity decisions. This power is fundamentally due to its exclusive accessibility to the new technology. Whereas firm M feels indifferent between the two possible equilibria, they are typically not equivalent in welfare terms (efficiency). This issue will be revisited soon.

Proposition 9 also indicates that the nonmonotonicity result is possible only when A, the productivity of technology 2, is sufficiently large. Technically, A determines the intersection of interval $[\left(\beta \frac{\alpha_2}{\alpha_1} + 1\right) k^*, \infty)$ and interval (x_1, x_2). When $A > \upsilon_1$, $(x_1, x_2) \subset [\left(\beta \frac{\alpha_2}{\alpha_1} + 1\right) k^*, \infty)$, which makes it possible to have a nonmonotonic relationship between the adoption time of the capital-intensive technology and the initial capital–labor ratio $\frac{K_0}{L}$ on interval $[\left(\beta \frac{\alpha_2}{\alpha_1} + 1\right) k^*, \infty)$, so part [1] of the proposition is obtained; when $A \in (\upsilon_2, \upsilon_1)$, we have $x_1 < \left(\beta \frac{\alpha_2}{\alpha_1} + 1\right) k^* < x_2$, so we obtain part [2], in which the nonmonotonicity result disappears; when $A < \upsilon_2$, the intersection set $(x_1, x_2) \cap [\left(\beta \frac{\alpha_2}{\alpha_1} + 1\right) k^*, \infty)$ is empty so Pattern BB always dominates Pattern CB from firm M's point of view, as with the result stated in part [3].

The intuition is that there are two competing effects when A increases. One is the substitution effect, making technology 2 more attractive when the capital–labor ratio is larger, so it tends to encourage capital saving in the first period and delay the adoption of technology 2. The other effect is the income effect: as the total income increases, it tends to discourage capital saving (encourage current production and consumption) by increasing the endogenous interest rate, so it facilitates the immediate adoption of technology 2, as long as the initial capital stock is sufficiently large. An increase in the initial capital–labor ratio would interact with these two effects. It turns out that only when A is sufficiently large is it possible to have the reversal of dominance between the substitution effect and the income effect as the initial capital–labor ratio increases.

Recall Lemma 8 shows that Pattern CC is the unique equilibrium, which is also socially efficient when the initial capital–labor ratio is sufficiently small ($\frac{K_0}{L} \leq \widetilde{\theta}_1 = \theta_1$). We also know that inefficiency arises when the initial capital–labor ratio is in the middle range when compared with the threshold values in Proposition 4. Is the market equilibrium socially efficient again when the initial capital–labor ratio becomes sufficiently large? In other words, is the nonmonotonic relationship between social efficiency and initial capital–labor ratio observed in the static model with private technology (see Lemma 3 and Proposition 4) preserved in the dynamic model?

To address these questions, first note that Lemma 8 indicates that Pattern BB is the unique market equilibrium when the initial capital–labor ratio is sufficiently large ($\frac{K_0}{L} > \max\{\left(\beta \frac{\alpha_2}{\alpha_1} + 1\right) k^*, x_2\}$), qualitatively the same pattern as the first-best case as shown in Proposition 4 (and Fig. 2). Now we check whether Pattern BB is socially efficient when $\frac{K_0}{L}$ is sufficiently large.

[7]

$$\widetilde{R}_{A2} = \frac{\xi^{\frac{\alpha_2-\alpha_1}{1-\alpha_1}} A^{\frac{1}{1-\alpha_1}} \left(\frac{K_0}{L}\right)^{\frac{\alpha_2-\alpha_1}{1-\alpha_1}} W_2}{(1+\beta)^{\frac{\alpha_2-\alpha_1}{1-\alpha_1}} \beta^{\frac{1-\alpha_2}{1-\alpha_1}} W_1},$$

$$\Pi_{A2} = \left(A^{\frac{1}{1-\alpha_1}} \left(\frac{\beta \xi K_0}{(1+\beta) L} \right)^{\frac{\alpha_2-\alpha_1}{1-\alpha_1}} - 1 \right) L W_2.$$

Both increase with $\frac{K_0}{L}$.

Proposition 10. *Suppose $\sigma = 1$, $\beta\xi \geq 1$, and $\frac{K_0}{L} \geq (\beta + 1)\, k^*$. When both technologies are free and public, only technology 2 is active in both periods (Pattern 2 in Table 1), and the corresponding (first-best) intertemporal capital allocation is*

$$E_1^{FB} = \frac{K_0}{1+\beta};\; E_2^{FB} = \frac{\beta\xi}{1+\beta} K_0.$$

When technology 2 is initially privately and exclusively accessible, the market equilibrium is Pattern BB, and the intertemporal capital allocation is given by

$$E_1^{BB} = \frac{K_0}{\beta\frac{\alpha_2}{\alpha_1} + 1};\, E_2^{BB} = \frac{\xi\beta\frac{\alpha_2}{\alpha_1}}{\beta\frac{\alpha_2}{\alpha_1} + 1} K_0.$$

Note that the intertemporal capital allocation for Pattern CB is given by

$$E_1^{CB} = \frac{K_0}{1+\beta};\, E_2^{CB} = \frac{\beta\xi}{1+\beta} K_0,$$

which is Pareto dominated by Pattern BB if and only if

$$\frac{K_0}{L} > \widetilde{\theta}_6 \equiv \left[\frac{\left(\beta\frac{\alpha_2}{\alpha_1} + 1\right)^{\alpha_2 + \beta\alpha_2}}{A\left(\frac{\alpha_2}{\alpha_1}\right)^{\beta\alpha_2} (1+\beta)^{\alpha_1 + \beta\alpha_2}} \right]^{\frac{1}{\alpha_2 - \alpha_1}}. \tag{38}$$

Proof. See Appendix F. ■

Note that the intertemporal capital allocations are identical for the first-best case and Pattern CB, but the first-period output is different because the adopted technology is different. The second-period output and technologies are the same in these two cases, although the market structures are different.

Observe that the adopted technology is identical for the first-best pattern and Pattern BB in each period when $\sigma = 1$, $\beta\xi \geq 1$ and $\frac{K_0}{L} \geq \max\{\left(\beta\frac{\alpha_2}{\alpha_1} + 1\right) k^*, x_2\}$. However, the intertemporal capital allocations are different for these two patterns, so the output levels are also different for each period. We conclude that the dynamic equilibrium Pattern CB is no longer socially efficient even when $\frac{K_0}{L}$ is arbitrarily large, different from the static case. More precisely, the private accessibility of technology 2 leads to excessive saving and capital overaccumulation ($E_1^{BB} < E_1^{FB}$). The reason is that positive monopoly rent is only available in the first period as technology 2 becomes public in the second period, so the intertemporal consumption smoothing requires more capital saving in period 1 to partly offset the missing monopoly rent, and hence the relatively low income in the second period.

Proposition 10 also helps rank the welfare associated with each equilibrium when there exist multiple equilibria. Consider the scenario for parts (1) and (2) in Proposition 9 (namely, $A > \upsilon_2$, $\sigma = 1$, $\beta\xi \geq 1$ and $\frac{K_0}{L} \in [\left(\beta\frac{\alpha_2}{\alpha_1} + 1\right) k^*, \infty)$). There are two equilibria when $\frac{K_0}{L} = x_2$. By (38), Pattern BB Pareto dominates Pattern CB if and only if $x_2 > \widetilde{\theta}_6$, which is true when A is sufficiently large.[8] The Pareto ranking would be the opposite if A.

4. Policy discussions

The previous analyses show that under certain circumstances, discrepancies exist between the first-best allocation and the market equilibrium with private technology, in which case the welfare-enhancing policy interventions become possible. Note that the two-factor markets are always perfectly competitive and well-functioning, but the factor price signals could be affected by the noncompetitive market structure in the final goods market via the general equilibrium effect, so factor price signals might no longer guide the socially efficient technology adoption. We have shown that new technology suffers adoption delay or insufficient utilization in some cases and premature adoption or overutilization in others. Moreover, an increase in the initial capital endowment may sometimes delay rather than facilitate the adoption of the new capital-intensive technology. We have also shown that noncompetitive market structures do not necessarily lead to social inefficiency, depending on the magnitude of factor endowment or whether factor prices are taken as exogenous by the firm with exclusive access to the new technology.

When inefficiency occurs, noncompetitive market structures appear to be the culprit, but the root of market inefficiency is the private access to the new technology. In this section, we discuss two possible sets of welfare-enhancing policies. One is to directly target private access to the new technology, and the other is to rectify the relative factor prices.

[8] More precisely, it occurs if and only if

$$A > \max\{\upsilon_2, \frac{\left[\left(\beta\frac{\alpha_2}{\alpha_1} + 1\right)^{(\alpha_2 + \beta\alpha_2) - \frac{(\alpha_2 - \alpha_1)^2}{(1-\alpha_1)}} \left(\frac{\alpha_2}{\alpha_1}\right)^{-\beta\alpha_2} (1+\beta)^{-(\alpha_1 + \beta\alpha_2)}\right]^{\frac{(1-\alpha_1)^2}{[1-\alpha_2][1-2\alpha_1+\alpha_2]}}}{\left[\frac{1}{2}\left((1+\beta) + \sqrt{(1+\beta)^2 - 4\beta\left[\frac{(1+\beta)}{\beta\xi\left(\beta\frac{\alpha_2}{\alpha_1}+1\right)}\right]^{\frac{\alpha_2-\alpha_1}{1-\alpha_1}}}\right)\right]^{\frac{(\alpha_2-\alpha_1)(1-\alpha_1)^2}{[1-\alpha_2][1-2\alpha_1+\alpha_2]}}}\}..$$

4.1. Enhancing accessibility to new technology

Given the existence of new technologies, one way to improve welfare is for government to enhance public access to these new technologies. For example, the government may support training or exchange programs for workers and entrepreneurs to facilitate knowledge learning and diffusion. The government could also help business associations for different industries in collecting and disseminating relevant information about technologies. The government could also relax unnecessary regulations and lower entry barriers for new firms, *etc.* Government typically does not necessarily know better than individual firms how to select the most promising technology, but it may be desirable for government to conduct timely surveys from a widely covered group of potential investors asking for opinions about what kind of technologies they are most interested in and then help collect and provide such information as a public service.[9]

When the new technology comes from abroad, it is useful to attract foreign direct investment by firms with access to it. The competition among these foreign firms would help make the new technology easier to access and more likely to become public, facilitating more efficient technology adoption. For more discussions on how FDI facilitates technology diffusion, see Harrison and Rodriguez-Clare (2009) and Wang (2013). Such policies may not only avoid efficient delays in technology adoption but also prevent premature adoption or overutilization of new technologies.

One caveat is that we take the existence of new technologies as exogenous for our analysis. However, to encourage *ex ante* R&D, it may be reasonable to authorize the innovator to enjoy monopoly rents for a certain period *ex post*, in which case noncompetitive market structures are not necessarily harmful.[10] Our paper implicitly assumes that the patent of the new technology has already expired, so imitation is free but subject to one period lag. Instead of proposing punishment for the *de jure* monopoly power (patent protection), we propose policy suggestions that try to break up *de facto* monopolistic or private access to the new technology.

4.2. Rectifying relative factor prices

Recall in the static model firm M, the monopolist of the new capital-intensive technology may produce less than the socially optimal amount because it fears that the rising rental price of capital would erode its monopoly profit when it increases its output. Thus the relative factor prices are distorted indirectly in general equilibrium, even though the factor markets themselves are perfect. In such circumstances, one welfare-enhancing approach is to rectify factor prices by subsidizing the capital cost of firm M to the extent that the welfare gain thanks to increased output more than offsets the welfare loss due to the factor price intervention. Another way is to impose labor income taxes on firms using the labor-intensive technology to facilitate the abandonment of the labor-intensive technology so that the production scale of the capital-intensive technology expands toward the socially optimal amount.

Recall in the dynamic model, the adoption of the capital-intensive technology may suffer inefficient delays (such as the case in Part (1) of Proposition 9 when $\frac{K_0}{L} \in (x_1, x_2)$), in which case the government could subsidize production with the capital-intensive technology in the first period or tax the production with the capital-intensive technology in the second period. Such policies could facilitate the quicker adoption of the new technology. To prevent premature adoption of the new technology (such as the case in Proposition 7), factor price rectification should work in the opposite direction. That is, capital should be taxed in firm M's production. To prevent over saving (such as the case in Proposition 10), production could be subsidized in the first period or taxed in the second.

The rationale for these interventions is not that the factor markets *per se* are imperfect, as is commonly argued in the literature on financial constraints. Instead, it is a second-best justification that uses factor markets intervention to partly undo the harm of monopoly due to private access to capital-intensive technology.

One may wonder what happens if the country is a small open economy with free international capital flow and an exogenous interest rate. In that case, firm M does not affect the interest rate in the market, yet it could still indirectly affect the wage rate, and hence the relative factor prices, by changing its output decision. Therefore, inefficiency may still exist in equilibrium, and some appropriate price intervention could improve welfare.

5. Conclusion

In this paper, we develop a simple dynamic general equilibrium model to explore technology adoption when the new technology is more capital-intensive and privately accessible. Private accessibility may result in a non-competitive market structure in the output market, which makes factor price signals no longer accurate to guide socially efficient technology adoption even though the factor markets themselves are perfect. We show that inefficiency arises only when the factor endowment structure lies in some intermediate-range and that allocation could be efficient even under monopoly. It is also shown that private accessibility may lead to delayed adoption in some cases and premature adoption in others. Moreover, an increase in initial capital endowment may

[9] Canda (2006) provides detailed cross-country case studies to illustrate how government plays an important welfare-enhancing role in technology adoption and diffusion in several developing countries. Lin and Monga (2011) provide a concrete six-step policy procedure for governments in developing countries to identify the right industrial target and facilitate industrial upgrading.

[10] In particular, Aghion et al. (2005) find that there exists an inverted-U relationship between market competition and innovation because too much competition may destroy the *ex ante* incentive of the potential investor who spends resources to acquire this new technology, similar to the issue of optimal patent duration design. Meanwhile, too concentrated market power also hurts innovation due to a lack of competition. So the antitrust law does not necessarily help or sometimes makes things even worse from the social efficiency point of view.

sometimes delay rather than facilitate the adoption of capital-intensive technology. Multiple equilibria may also arise in the dynamic economy. Welfare-enhancing policies are discussed.

Several avenues seem interesting for future research. One is to introduce productivity heterogeneity for the new technology in the spirit of Jones (2005). Another direction is to introduce multiple players (firms) that are all accessible to the new technology and to embed their strategic interaction into the dynamic general equilibrium framework (Bolton and Farrell, 1990; Ederington and McCalman, 2009). A third direction is to incorporate the endogenous creation of new technologies or technological progress into the analysis. We conjecture that technologies more consistent with factor endowment advance faster because newly innovated products more consistent with factor endowment are more cost-efficient and hence more profitable, giving firms incentives to invest more in R&D for those technologies. It would imply that labor-augmenting (capital-augmenting) technological progress would be more likely in labor-abundant (capital-abundant) economies. However, if a developing (labor-abundant) country grows faster than a developed (capital-abundant) country, the factor endowment structure of the former becomes increasingly closer to that of the latter, so it might optimally switch to adopting the relatively capital-intensive technologies invented in the developed economy instead of relying on its own labor-augmenting technological progress, and the potential switching decision depends on the adoption cost, which in turn is affected by whether the patent has already expired or not. We leave the detailed analyses for future research.

CRediT authorship contribution statement

Yong Wang: Conceptualization, Methodology, Writing – original draft, Writing – review & editing, Validation.

Data availability

No data was used for the research described in the article.

Appendix A. Production department takes factor prices as endogenous

In Scenario 2, we assume that the production department of firm M is the price taker at the factor markets, although the output decision of firm M internalizes the impact of its output decision on factor prices. It simplifies the analysis because the factor markets are still perfectly competitive. If, however, firm M fully takes advantage of its role as a monopsonist, both (15) and (20) are invalid as its decision on inputs (hence unit cost) no longer takes factor prices as exogenous.

Suppose firm M decides to serve only a fraction of the market, that is, both technologies are adopted in the market, the firm M solves the following problem:

$$\max_{P,W,R,K_2,L_2} \Pi = PAK_2^{\alpha_2} L_2^{*1-\alpha_2} - RK_2 - WL_2 \tag{39}$$

subject to

$$P \leq \mu_1(W,R); 0 < K_2 < K; 0 < L_2 < L,$$

and

$$\frac{K-K_2}{L-L_2} = \frac{K_1}{L_1} = \frac{\alpha_1}{1-\alpha_1}\frac{W}{R}, \tag{40}$$

where $\mu_1(W,R)$ is given by (6), the first equation in (40) comes from the factor market clearing conditions, and the second equation in (40) comes from the fact that a competitive fringe of firms with technology 1 are still price takers in the factor markets. Since both technologies are active, their prices must be equal:

$$P = \mu_1(W,R) = \frac{R^{\alpha_1} W^{1-\alpha_1}}{A_1 \alpha_1^{\alpha_1} \left(1-\alpha_1\right)^{1-\alpha_1}} \tag{41}$$

Without loss of generality, we could price everything in terms of wage, so we rewrite (39) as

$$\max_{P,W,R,K_2,L_2} \frac{\Pi}{W} = \frac{P}{W} AK_2^{\alpha_2} L_2^{1-\alpha_2} - \frac{R}{W} K_2 - L_2.$$

Substituting (40) and (41) into the above objective function, we obtain

$$\max_{K_2,L_2} \frac{\Pi}{W} = \frac{AK_2^{\alpha_2} L_2^{1-\alpha_2}}{\left(1-\alpha_1\right)} \left(\frac{L-L_2}{K-K_2}\right)^{\alpha_1} - \frac{\alpha_1}{1-\alpha_1}\frac{L-L_2}{K-K_2} K_2 - L_2$$

The first-order condition with respect to K_2 is

$$\left[\frac{\alpha_2\left(K-K_2\right)+\alpha_1 K_2}{\alpha_1 K}\right] AK_2^{\alpha_2-1}\left(K-K_2\right)^{1-\alpha_1} = \frac{\left(L-L_2\right)^{1-\alpha_1}}{L_2^{1-\alpha_2}}, \tag{42}$$

which implies that K_2 strictly increases with L_2. In addition, (42) can be rewritten as

$$\left[\frac{\alpha_2\left(K-K_2\right)+\alpha_1 K_2}{\alpha_1 K}\right] \frac{\left(K-K_2\right)}{K_2} = \frac{\left(K-K_2\right)^{\alpha_1}\left(L-L_2\right)^{1-\alpha_1}}{AK_2^{\alpha_2} L_2^{1-\alpha_2}}$$

$$\frac{\alpha_1 \left(K - K_2\right)^{\alpha_1 - 1} \left(L - L_2\right)^{1-\alpha_1}}{\alpha_2 A K_2^{\alpha_2 - 1} L_2^{1-\alpha_2}} = \frac{K - \frac{\alpha_2 - \alpha_1}{\alpha_2} K_2}{K},$$

where the left-hand side is the ratio of the marginal product of capital in technology 1 to that of technology 2, and the right-hand side of the equation is strictly smaller than one. It implies that the production deviates from the first best in that too much capital is allocated to technology 1. In other words, firm M purposefully reduces its demand for capital to depress the rental price of capital, as its technology is more capital-intensive.

The first-order condition with respect to L_2 for firm M is

$$A L_2^{-\alpha_2} \left(L - L_2\right)^{\alpha_1 - 1} \left[\left(1 - \alpha_2\right)\left(L - L_2\right) - \alpha_1 L_2\right] = \frac{\left(1 - \alpha_1\right) K - K_2}{K_2^{\alpha_2} \left(K - K_2\right)^{1-\alpha_1}}. \tag{43}$$

We know that, when both technologies are active, K_2 and L_2 are jointly determined by (42) and (43). All other endogenous variables, including K_1, L_1, $\frac{R}{W}$, are thus easily derived.

Appendix B. Proof of Lemma 3

Let $\widetilde{k}_1$ and $\widetilde{k}_2$ denote the equilibrium capital–labor ratios for the two technologies. From the factor market clearing conditions, we obtain the total output for each technology:

$$Q_1^m = \Phi(\widetilde{k}_1, \widetilde{k}_2) \equiv \left(\frac{\widetilde{k}_2 L - K}{\widetilde{k}_2 - \widetilde{k}_1}\right)\left(\widetilde{k}_1\right)^{\alpha_1}; \; Q_2^m = \Psi(\widetilde{k}_1, \widetilde{k}_2) \equiv A\left(\frac{K - \widetilde{k}_1 L}{\widetilde{k}_2 - \widetilde{k}_1}\right)\left(\widetilde{k}_2\right)^{\alpha_2},$$

where $\widetilde{k}_1$ and $\widetilde{k}_2$ are given by (16) and (17), respectively and $\frac{R}{W}$ is determined by (22). In the competitive equilibrium, we have

$$Q_1^c = \Phi(k_1^*, k_2^*); \; Q_2^c = \Psi(k_1^*, k_2^*),$$

where k_1^* and k_2^* are given by (1) and (2). Since (16)–(17) always hold, independent of the market structure in the goods market, $\frac{R}{W} < \psi$ implies $\widetilde{k}_1 > k_1^*$ and $\widetilde{k}_2 > k_2^*$. Moreover, $\widetilde{k}_2 > k > \widetilde{k}_1$. Consequently. $Q_1^m > Q_1^c$ and $Q_2^m < Q_2^c$. This is because function $\Phi(\cdot,\cdot)$ strictly increases in both arguments while function $\Psi(\cdot,\cdot)$ strictly decreases in both arguments. Resource allocation is distorted compared with competitive equilibrium, so $Q_1^m + Q_2^m < Q_1^c + Q_2^c$.

Appendix C. This is to prove Proposition 5

Pattern 1: Only Technology 1 in both periods Establish the Lagrangian:

$$\mathcal{L} = \frac{\left[E_1^{\alpha_1} L^{1-\alpha_1}\right]^{1-\sigma} - 1}{1-\sigma} + \beta \frac{\left[E_2^{\alpha_1} L^{1-\alpha_1}\right]^{1-\sigma} - 1}{1-\sigma} + \lambda\left[\xi(K_0 - E_1) - E_2\right],$$

which yields the following two first-order conditions relative to E_1 and E_2, respectively:

$$\begin{aligned} \alpha_1 \left[E_1^{\alpha_1} L^{1-\alpha_1}\right]^{-\sigma} E_1^{\alpha_1 - 1} L^{1-\alpha_1} &= \lambda\xi, \\ \beta\alpha_1 \left[E_2^{\alpha_1} L^{1-\alpha_1}\right]^{-\sigma} E_2^{\alpha_1 - 1} L^{1-\alpha_1} &= \lambda. \end{aligned}$$

We obtain

$$E_1 = \frac{K_0}{1 + \xi^{-1}(\beta\xi)^{\frac{1}{-\alpha_1+1+\alpha_1\sigma}}}; \; E_2 = K_0 \frac{(\beta\xi)^{\frac{1}{-\alpha_1+1+\alpha_1\sigma}}}{1 + \xi^{-1}(\beta\xi)^{\frac{1}{-\alpha_1+1+\alpha_1\sigma}}}.$$

Since $\beta\xi > 1$ and $\sigma \in [0,1]$, to ensure $E_1 \le k_1^* L$ and $E_2 \le k_1^* L$, we must have

$$\frac{K_0}{L} \le \theta_1 k_1^*, \tag{44}$$

where k_1^* is given by (16) and θ_1 is defined as

$$\theta_1 \equiv \frac{1 + \xi^{-1}(\beta\xi)^{\frac{1}{-\alpha_1+1+\alpha_1\sigma}}}{(\beta\xi)^{\frac{1}{-\alpha_1+1+\alpha_1\sigma}}}.$$

In other words, technology 1 alone is adopted in both periods if and only if (44) holds.

Pattern 2: Technology 2 in both periods Following the same method, we have:

$$E_1 = \frac{K_0}{1 + \xi^{-1}(\beta\xi)^{\frac{1}{-\alpha_2+1+\alpha_2\sigma}}}; \; E_2 = K_0 \frac{(\beta\xi)^{\frac{1}{-\alpha_2+1+\alpha_2\sigma}}}{1 + \xi^{-1}(\beta\xi)^{\frac{1}{-\alpha_2+1+\alpha_2\sigma}}},$$

To ensure E_1 and E_2 larger than $k_2^* L$, we require

$$\frac{K_0}{L} \ge \theta_6 k_1^*, \tag{45}$$

where k_2^* is given by (17) and θ_6 is defined as

$$\theta_6 \equiv \frac{\alpha_2\left(1-\alpha_1\right)}{\alpha_1\left(1-\alpha_2\right)}\left(1+\xi^{-1}(\beta\xi)^{\frac{1}{-\alpha_2+1+\alpha_2\sigma}}\right).$$

In other words, technology 2 alone is adopted in both periods if and only if (45) holds.

Pattern 3: Technologies 1 and 2 in both periods We can derive from the first-order conditions that

$$E_1 = \frac{\left[1-(\beta\xi)^{\frac{1}{\sigma}}\right](1-\alpha_1)k_1^*L+\alpha_1\xi K_0}{[(\beta\xi)^{\frac{1}{\sigma}}+\xi]\alpha_1},$$
$$E_2 = \xi(\frac{\alpha_1(\beta\xi)^{\frac{1}{\sigma}}K_0-\left[1-(\beta\xi)^{\frac{1}{\sigma}}\right](1-\alpha_1)\left(k_1^*\right)L}{[(\beta\xi)^{\frac{1}{\sigma}}+\xi]\alpha_1}).$$

To ensure $k_1^*L < E_1, E_2 < k_2^*L$, we must have

$$\theta_3 k_1^* < \frac{K_0}{L} < \theta_4 k_1^*,$$

where

$$\theta_3 \equiv \frac{(\beta\xi)^{\frac{1}{\sigma}}+\xi\alpha_1-(1-\alpha_1)}{\alpha_1\xi};\theta_4 \equiv \left[\frac{1}{\xi}+\frac{(\beta\xi)^{-\frac{1}{\sigma}}-\left(1-\alpha_2\right)}{\alpha_2}\right]\frac{\alpha_2\left(1-\alpha_1\right)}{\alpha_1\left(1-\alpha_2\right)}.$$

The nonemptiness of K_0 further requires $\theta_3 < \theta_4$, or

$$(\beta\xi) < \left[\frac{\left(1-\alpha_1\right)}{\left(1-\alpha_2\right)}\right]^{\sigma}. \tag{46}$$

That is, ξ has to be sufficiently small.

Pattern 4: Technology 1 in period 1, Technologies 1 and 2 in period 2 The Euler equation is

$$\left(A_1\left(k_1^*\right)^{\alpha_1}-\frac{A_2\left(k_2^*\right)^{\alpha_2}-A_1\left(k_1^*\right)^{\alpha_1}}{k_2^*-k_1^*}k_1^*\right)L+\frac{A_2\left(k_2^*\right)^{\alpha_2}-A_1\left(k_1^*\right)^{\alpha_1}}{k_2^*-k_1^*}\xi(K_0-E_1)$$
$$=\left[\frac{\beta\xi\left[A_2\left(k_2^*\right)^{\alpha_2}-A_1\left(k_1^*\right)^{\alpha_1}\right]}{\alpha_1 L^{(1-\alpha_1)(1-\sigma)}\left(k_2^*-k_1^*\right)}\right]^{\frac{1}{\sigma}}E_1^{\frac{\sigma\alpha_1-\alpha_1+1}{\sigma}}.$$

To ensure $E_1 \le k_1^*L$, we need

$$\frac{K_0}{L} \le \theta_3 k_1^*. \tag{47}$$

On the other hand,

$$\left(A_1\left(k_1^*\right)^{\alpha_1}-\frac{A_2\left(k_2^*\right)^{\alpha_2}-A_1\left(k_1^*\right)^{\alpha_1}}{k_2^*-k_1^*}k_1^*\right)L+\frac{A_2\left(k_2^*\right)^{\alpha_2}-A_1\left(k_1^*\right)^{\alpha_1}}{k_2^*-k_1^*}E_2$$
$$=\left[\frac{\beta\xi\left[A_2\left(k_2^*\right)^{\alpha_2}-A_1\left(k_1^*\right)^{\alpha_1}\right]}{\alpha_1 L^{(1-\alpha_1)(1-\sigma)}\left(k_2^*-k_1^*\right)}\right]^{\frac{1}{\sigma}}\left(K_0-\frac{E_2}{\xi}\right)^{\frac{\sigma\alpha_1-\alpha_1+1}{\sigma}},$$

To ensure $k_1^*L < E_2 < k_2^*L$, we get

$$\theta_1 k_1^* < \frac{K_0}{L} < \theta_2 k_1^*, \tag{48}$$

where

$$\theta_2 \equiv [\beta\xi]^{\frac{1}{-\sigma\alpha_1+\alpha_1-1}}\left[\frac{1-\alpha_1}{1-\alpha_2}\right]^{\frac{\sigma}{\sigma\alpha_1-\alpha_1+1}}+\frac{\alpha_2\left(1-\alpha_1\right)}{\alpha_1\left(1-\alpha_2\right)\xi}.$$

The nonemptiness for the set of K_0 requires $\theta_1 < \theta_2$, or equivalently,

$$[\beta\xi]^{\frac{1}{-\sigma\alpha_1+\alpha_1-1}} < \frac{[\beta\xi]^{\frac{1}{\sigma}}-1}{\alpha_1\xi}+1,$$

which must hold because $\beta\xi > 1$. Also we require

$$\theta_1 < \theta_3,$$

or equivalently,

$$\alpha_1\xi < (\beta\xi)^{\frac{1}{-\alpha_1+1+\alpha_1\sigma}}\left[(\beta\xi)^{\frac{1}{\sigma}} + \xi\alpha_1 - 1\right],$$

which automatically holds. In summary, the equilibrium demonstrates this industrial pattern if and only if (47) and (48) hold.

Pattern 5: Technology 1 in period 1, Technology 2 in period 2 The Euler equation is

$$\beta\xi\alpha_2\left[\xi(K_0 - E_1)\right]^{\alpha_2-1-\sigma\alpha_2}\left(AL^{1-\alpha_2}\right)^{1-\sigma} = \alpha_1 E_1^{\alpha_1-1-\alpha_1\sigma}L^{(1-\alpha_1)(1-\sigma)}.$$

So $E_1 \le k_1^* L$ implies

$$\beta\xi\alpha_2\left[\xi(K_0 - k_1^* L)\right]^{\alpha_2-1-\sigma\alpha_2}\left(AL^{1-\alpha_2}\right)^{1-\sigma} \ge \alpha_1\left(k_1^* L\right)^{\alpha_1-1-\alpha_1\sigma}L^{(1-\alpha_1)(1-\sigma)},$$

which is equivalent to

$$\frac{K_0}{L} \le \theta_5 k_1^*,$$

where

$$\theta_5 \equiv \left(\frac{\alpha_1}{\alpha_2}\right)^{-1}\left[\beta\xi^{(1-\sigma)\alpha_2}\left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{(1-\alpha_2)((1-\sigma))}\right]^{\frac{1}{1-\alpha_2+\sigma\alpha_2}} + 1.$$

On the other hand

$$\beta\xi\alpha_2 E_2^{\alpha_2-1-\sigma\alpha_2}\left(AL^{1-\alpha_2}\right)^{1-\sigma} = \alpha_1\left(K_0 - \frac{E_2}{\xi}\right)^{\alpha_1-1-\alpha_1\sigma}L^{(1-\alpha_1)(1-\sigma)},$$

so $E_2 \ge k_2^* L$ implies

$$\beta\xi\alpha_2\left(k_2^* L\right)^{\alpha_2-1-\sigma\alpha_2}\left(AL^{1-\alpha_2}\right)^{1-\sigma} \ge \alpha_1\left(K_0 - \frac{Lk_2^*}{\xi}\right)^{\alpha_1-1-\alpha_1\sigma}L^{(1-\alpha_1)(1-\sigma)},$$

which is reduced to

$$\frac{K_0}{L} \ge \theta_2 k_1^*.$$

In summary, we must have

$$\theta_2 k_1^* \le \frac{K_0}{L} \le \theta_5 k_1^*,$$

the nonemptiness of which requires $\theta_2 \le \theta_5$, or equivalently

$$\left[(\beta\xi)^{-1}\left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{\sigma}\right]^{\frac{1}{\sigma\alpha_1-\alpha_1+1}} \le \frac{\alpha_2\left(1-\alpha_1\right)}{\alpha_1\left(1-\alpha_2\right)\xi}\left\{\left[(\beta\xi)^{-1}\left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{\sigma}\right]^{-\frac{1}{1-\alpha_2+\sigma\alpha_2}} - 1\right\} + 1, \tag{49}$$

which holds only if (46) does not hold. In other words, Pattern 3 and Pattern 5 are incompatible with each other.

Pattern 6: Technologies 1 and 2 in period 1, Technology 2 in period 2:

$$\beta\xi\alpha_2\left[AE_2^{\alpha_2}L^{1-\alpha_2}\right]^{-\sigma}AE_2^{\alpha_2-1}L^{1-\alpha_2}$$
$$= \left[\begin{array}{c}\left(A_1\left(k_1^*\right)^{\alpha_1} - \frac{A_2\left(k_2^*\right)^{\alpha_2} - A_1\left(k_1^*\right)^{\alpha_1}}{k_2^*-k_1^*}k_1^*\right)L \\ + \frac{A_2\left(k_2^*\right)^{\alpha_2} - A_1\left(k_1^*\right)^{\alpha_1}}{k_2^*-k_1^*}E_1\end{array}\right]^{-\sigma}\frac{A_2\left(k_2^*\right)^{\alpha_2} - A_1\left(k_1^*\right)^{\alpha_1}}{k_2^* - k_1^*}.$$

From $k_1^* L < E_1 < k_2^* L$, we obtain

$$\theta_6 k_1^* > \frac{K_0}{L} > \theta_5 k_1^*. \tag{50}$$

To ensure $\theta_6 > \theta_5$, we require

$$\frac{\alpha_2\left(1-\alpha_1\right)}{\alpha_1\left(1-\alpha_2\right)}\left(1 + \xi^{-1}(\beta\xi)^{\frac{1}{-\alpha_2+1+\alpha_2\sigma}}\right) > \frac{\alpha_2}{\alpha_1}\left[\beta\xi^{(1-\sigma)\alpha_2}\left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{(1-\alpha_2)((1-\sigma))}\right]^{\frac{1}{1-\alpha_2+\sigma\alpha_2}} + 1,$$

which must be true because $\frac{\alpha_2(1-\alpha_1)}{\alpha_1(1-\alpha_2)} > 1$ and

$$\frac{\left(1-\alpha_1\right)}{\left(1-\alpha_2\right)}\xi^{-1}(\beta\xi)^{\frac{1}{-\alpha_2+1+\alpha_2\sigma}} > \left[\beta\xi^{(1-\sigma)\alpha_2}\left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{(1-\alpha_2)(1-\sigma)}\right]^{\frac{1}{1-\alpha_2+\sigma\alpha_2}}.$$

On the other hand $E_2 \geq k_2^* L$ requires

$$\frac{K_0}{L} \geq \theta_4 k_1^*. \tag{51}$$

To ensure $\theta_6 > \theta_4$, we require

$$(\beta\xi)^{\frac{1}{-\alpha_2+1+\sigma\alpha_2}} > \left[\frac{(\beta\xi)^{-\frac{1}{\sigma}} - 1}{\alpha_2}\right]\xi + 1,$$

which automatically holds because $\beta\xi > 1$. In summary, the equilibrium has this pattern if and only if (50) and (51) hold.

Appendix D. Proof of Lemma 6

Proof. Pattern **AC.** $\frac{E_2}{L} \in (0, k_1^*]$ so that only technology 1 is operated in period 2. Note that

$$\frac{P_1}{R_1} = \frac{\left(\Gamma(\frac{E_1}{L})\right)^{\alpha_1-1}}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}}; \quad \frac{P_2}{R_2} = \frac{\left(\frac{R_2}{W_2}\right)^{\alpha_1-1}}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}}; \quad \frac{W_2}{R_2} = \frac{(1-\alpha_1)E_2}{\alpha_1 L}$$

(36) becomes

$$\beta\xi\left(\frac{\xi^{\alpha_1}(\frac{K_0}{L} - \frac{E_1}{L})^{\alpha_1}}{y_1(\frac{E_1}{L})}\right)^{-\sigma} = \left(\frac{(1-\alpha_1)}{\alpha_1}\xi(\frac{K_0}{L} - \frac{E_1}{L})\Gamma(\frac{E_1}{L})\right)^{1-\alpha_1} \tag{52}$$

which can uniquely pin down $\frac{E_1}{L}$. Clearly $\frac{R_2}{W_2} > \Gamma(\frac{E_1}{L})$, so $\frac{P_1}{R_1} > \frac{P_2}{R_2}$; thus, the RHS of (52) is smaller than one, which requires $\frac{\xi^{\alpha_1}(\frac{K_0}{L} - \frac{E_1}{L})^{\alpha_1}}{y_1(\frac{E_1}{L})} > 1$ when $\beta\xi > 1$ and $\sigma \in (0, 1]$. But we must have $y_1(\frac{E_1}{L}) > \xi^{\alpha_1}(\frac{K_0}{L} - \frac{E_1}{L})^{\alpha_1}$ because of the adoption pattern. Therefore, this pattern is impossible.

Pattern **BC.** Only technology 1 in period 2.

$$\begin{aligned} P_2/R_2 &= \frac{\left(\frac{W_2}{R_2}\right)^{1-\alpha_1}}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}} = \frac{\left(\frac{E_2}{L}\frac{1-\alpha_1}{\alpha_1}\right)^{1-\alpha_1}}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}} \\ \frac{W_2}{R_2} &= \frac{E_2}{L}\frac{1-\alpha_1}{\alpha_1} \end{aligned}$$

(36) becomes

$$\beta\xi = \left(\frac{\left[\xi(K_0 - E_1)\right]^{\alpha_1} L^{1-\alpha_1}}{A E_1^{\alpha_2} L^{1-\alpha_2}}\right)^{\sigma}\left(\frac{\xi(K_0 - E_1)}{E_1}\frac{1-\alpha_1}{\alpha_1}\frac{\alpha_2}{1-\alpha_2}\right)^{1-\alpha_1}, \tag{53}$$

which uniquely determines E_1. We require $E_1^* \geq k^* L$ and $E_2^* = \xi(K_0 - E_1) \leq k_1^* L$, which makes the right-hand side of (53) smaller than one, contradicting $\beta\xi > 1$.

Pattern **BA.** Both technologies in period 2 (that is, $\frac{\xi(K_0 - E_1^*)}{L} \in (k_1^*, k_2^*)$)

$$\begin{aligned} P_1/R_1 &= \frac{1}{\left(\frac{\alpha_2}{1-\alpha_2}\frac{L}{E_1}\right)^{1-\alpha_1}(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}}; \\ P_2/R_2 &= \frac{1}{\left(\left[A\frac{\alpha_2^{\alpha_2}(1-\alpha_2)^{(1-\alpha_2)}}{\alpha_1^{\alpha_1}(1-\alpha_1)^{(1-\alpha_1)}}\right]^{\frac{1}{\alpha_2-\alpha_1}}\right)^{1-\alpha_1}(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}}; \\ \frac{P_2/R_2}{P_1/R_1} &= \frac{\left(\frac{\alpha_2}{1-\alpha_2}\frac{L}{E_1}\right)^{1-\alpha_1}}{\left(\left[A\frac{\alpha_2^{\alpha_2}(1-\alpha_2)^{(1-\alpha_2)}}{\alpha_1^{\alpha_1}(1-\alpha_1)^{(1-\alpha_1)}}\right]^{\frac{1}{\alpha_2-\alpha_1}}\right)^{1-\alpha_1}} < 1. \\ \beta\xi\left(\frac{C_2}{C_1}\right)^{-\sigma} &= \frac{P_2/R_2}{P_1/R_1}. \end{aligned}$$

Consider the last equation (Euler equation). Whenever $\sigma \in [0, 1]$, $LHS > 1$ because $\frac{C_2}{C_1} < 1$ and $\beta\xi > 1$; whereas the $RHS < 1$, a contradiction.

Appendix E. Proof for Proposition 7

Pattern CA: Only technology 1 in period 1 and both technologies in period 2

$$P_2 = \frac{R_2^{\alpha_1} W_2^{1-\alpha_1}}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}}; \quad \frac{P_2}{R_2} = \frac{\left(\frac{R_2}{W_2}\right)^{\alpha_1-1}}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}}; \quad \frac{R_2}{W_2} = \Gamma(\frac{E_2}{L}).$$

Euler Eq. (36) becomes

$$\beta\xi\left(\frac{\widetilde{G}(\xi(K_0-E_1),L)}{E_1^{\alpha_1}L^{1-\alpha_1}}\right)^{-\sigma} = \frac{P_2/R_2}{P_1/R_1} = \left(\frac{W_2}{R_2}\frac{R_1}{W_1}\right)^{1-\alpha_1},$$

where function $\widetilde{G}(.,.)$ is defined in (23). This implies (37), which uniquely determines E_1 if it exists, because the LHS increases with E_1 while the RHS decreases with it. Suppose $\sigma = 1$ and $\frac{K_0}{L} = \frac{1+\beta}{\beta\xi}k_1^*$. When $E_1 \to 0$, LHS→ 0; RHS→ ∞; When $\frac{\xi(K_0-E_1)}{L} \downarrow k_1^*$ (or equivalently, $\frac{E_1}{L} \uparrow \frac{k_1^*}{\beta\xi}$), $LHS \to \beta\xi\left(\frac{\left(k_1^*\right)^{\alpha_1}L}{\left(\frac{k_1^*}{\beta\xi}\right)^{\alpha_1}L}\right)^{-1} = (\beta\xi)^{1-\alpha_1}$ while $RHS = \left(\frac{1}{\Gamma(\frac{\xi(K_0-E_1)}{L})}\frac{\alpha_1}{1-\alpha_1}\frac{L}{E_1}\right)^{1-\alpha_1} \to (\beta\xi)^{1-\alpha_1}$. In other words, whenever $\frac{\xi(K_0-E_1)}{L} > k_1^*$, we always have $LHS < RHS$, implying that no solution to (37) exists. Now suppose $\frac{K_0}{L} = \theta_1$ (given by (30)) and $\sigma < 1$. When $E_1 \to 0$, LHS→ 0; RHS→ ∞. When $\frac{\xi(K_0-E_1)}{L} \downarrow k_1^*$ (or equivalently, $\frac{E_1}{L} \uparrow (\beta\xi)^{-\frac{1}{-\alpha_1+1+\alpha_1\sigma}}k_1^*$), $LHS \to (\beta\xi)^{\frac{\alpha_1+1}{-\alpha_1+1+\alpha_1\sigma}}$ and $RHS \to (\beta\xi)^{\frac{1-\alpha_1}{-\alpha_1+1+\alpha_1\sigma}}$. Thus $LHS > RHS$, so by the Mean Value Theorem, there exists a unique solution $\frac{E_1}{L}$, denoted as θ_0, which falls on $(0,(\beta\xi)^{-\frac{1}{-\alpha_1+1+\alpha_1\sigma}}k_1^*)$ such that (37) is satisfied.

To support Pattern CA, we must require $\frac{\xi(K_0-E_1)}{L} \in (k_1^*,k^*)$, so $\frac{k_1^*}{\xi} < \frac{K_0}{L} < 2\frac{k^*}{\xi}$. In particular, when $\sigma = 1$, (37) becomes

$$\beta\xi y^{-1}(\frac{\xi(K_0-E_1)}{L})\frac{E_1}{L} = \left(\frac{1}{\Gamma(\frac{\xi(K_0-E_1)}{L})}\frac{\alpha_1}{1-\alpha_1}\right)^{1-\alpha_1},$$

so

$$\left[\frac{\alpha_2(1-\alpha_1)}{\alpha_1\alpha_2-\alpha_1^2+\alpha_2-\alpha_2^2}\right]^{\frac{1+\alpha_2-\alpha_1}{\alpha_2-\alpha_1}}\frac{(1-\alpha_1)k_1^*}{(1-\alpha_2)\beta\xi} > \frac{E_1}{L} > \frac{k_1^*}{\beta\xi}.$$

On the other hand,

$$\frac{k_1^*}{\xi} + \frac{E_1}{L} < \frac{K_0}{L} < \frac{k^*}{\xi} + \frac{E_1}{L},$$

so we conclude

$$\begin{aligned}\widetilde{\theta}_1 &\equiv \frac{k_1^*}{\xi} + \frac{k_1^*}{\beta\xi} < \frac{K_0}{L} < \widetilde{\theta}_2 \\ &\equiv \frac{k^*}{\xi} + \frac{\alpha_1(1-\alpha_1)}{\alpha_1\alpha_2-\alpha_1^2+\alpha_2-\alpha_2^2}\frac{k^*}{\beta\xi}.\end{aligned}$$

The related present discounted profit is

$$\begin{aligned}\frac{\Pi_{CA}}{\widetilde{R}} = &\frac{A\left(\frac{\alpha_2}{1-\alpha_2}\right)^{\alpha_2}(1-\alpha_1)(1-\alpha_2)}{\alpha_2-\alpha_1}\left[\Gamma(\frac{E_2}{L})\frac{E_2}{L} - \frac{\alpha_1}{1-\alpha_1}\right]\cdot \\ &\left[\frac{\Gamma(\frac{E_2}{L})^{\alpha_1-\alpha_2}}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}} - \frac{1}{A(\alpha_2)^{\alpha_2}(1-\alpha_2)^{1-\alpha_2}}\right]\frac{1}{\xi\Gamma(\frac{E_2}{L})}\frac{\alpha_1}{1-\alpha_1}\frac{L}{E_1}LW_1.\end{aligned} \tag{54}$$

This implies that, for any given K_0, $\frac{\Pi_{CA}}{\widetilde{R}}$ strictly increases when E_2 increases and E_1 decreases, so

$$\begin{aligned}\frac{\Pi_{CA}}{\widetilde{R}} < &\frac{A\left(\frac{\alpha_2}{1-\alpha_2}\right)^{\alpha_2}(1-\alpha_1)(1-\alpha_2)}{\alpha_2-\alpha_1}\left[\Gamma(k^*)k^* - \frac{\alpha_1}{1-\alpha_1}\right]\cdot \\ &\left[\frac{\Gamma(k^*)^{\alpha_1-\alpha_2}}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}} - \frac{1}{A(\alpha_2)^{\alpha_2}(1-\alpha_2)^{1-\alpha_2}}\right]\frac{1}{\xi\Gamma(k^*)}\frac{\alpha_1}{1-\alpha_1}\frac{1}{\frac{K_0}{L}-\frac{k^*}{\xi}}LW_1\end{aligned}$$

In particular, when $\sigma = 1$, (37) becomes

$$\beta\xi\widehat{Y}^{-1}(\xi(K_0-E_1),L,\Gamma(\frac{\xi(K_0-E_1)}{L})) = \frac{1}{E_1}\left(\frac{1}{\Gamma(\frac{\xi(K_0-E_1)}{L})}\frac{\alpha_1}{1-\alpha_1}\right)^{1-\alpha_1}$$

$$\Pi_{B1a} = \left[\left(A\left[\frac{K_0/L}{\beta\frac{\alpha_2}{\alpha_1}+1}\right]^{\alpha_2-\alpha_1}\right)^{\frac{1}{1-\alpha_1}} - 1\right] W_1 L.$$

$$\Gamma(k^*) = \left[\frac{\left(1-\alpha_1\right)^{2-\alpha_1}\alpha_1^{\alpha_1}}{A\alpha_2^{\alpha_2-1}\left(1-\alpha_2\right)^{(1-\alpha_2)}\left[\alpha_1\alpha_2-\alpha_1^2+\alpha_2-\alpha_2^2\right]}\right]^{\frac{1}{\alpha_1-\alpha_2}}$$

Appendix F. Proof for Lemma 8

F.1. Option 1: Immediate adoption of technology 2

In period 1, technology 2 can be either operated together with technology 1 (Possibility A) or solely operated (Possibility B). Since the market is perfectly competitive in the second period, we have $\Pi_2 = 0$. By Lemma 6, in period 2, either only technology 2 is adopted or both technologies.

Possibility A for Option 1. Both technologies are operated in period 1.

In this case, as suggested by (21), we have

$$\begin{aligned} P_1/R_1 &= \mu_1(W_1,R_1)/R_1 = \frac{\left(\frac{R_1}{W_1}\right)^{\alpha_1-1}}{\left(\alpha_1\right)^{\alpha_1}\left(1-\alpha_1\right)^{1-\alpha_1}} \\ &= \frac{\left(\Gamma(\frac{E_1}{L})\right)^{\alpha_1-1}}{\left(\alpha_1\right)^{\alpha_1}\left(1-\alpha_1\right)^{1-\alpha_1}}, \end{aligned}$$

where function $\Gamma(\frac{E_1}{L})$ is defined as the implicit solution of $\frac{R_1}{W_1}$ as a function of $\frac{E_1}{L}$ in (22) with k replaced by $\frac{E_1}{L}$. Observe $\Gamma'(\frac{E_1}{L}) < 0$ but $\frac{E_1}{L}\Gamma(\frac{E_1}{L})$ is a strictly increasing function of $\frac{E_1}{L}$. To determine $\frac{E_1}{L}$, note that the total output in period 1 is $\widetilde{G}(E_1, L)$, defined in (23). In period 2, both technologies are freely available, and the market is perfectly competitive.

Pattern AB: both technologies in period 1 and only technology 2 in period 2

$$\begin{aligned} P_2/R_2 &= \mu_2(W_2,R_2)/R_2 = \frac{\left(\frac{W_2}{R_2}\right)^{1-\alpha_2}}{A\left(\alpha_2\right)^{\alpha_2}\left(1-\alpha_2\right)^{1-\alpha_2}} \\ \frac{W_2}{R_2} &= \frac{\left(1-\alpha_2\right)E_2}{\alpha_2 L} = \frac{\left(1-\alpha_2\right)\xi(K_0-E_1)}{\alpha_2 L}. \end{aligned}$$

In equilibrium, (36) implies

$$\beta\xi A^{1-\sigma}\alpha_2\Gamma^{\alpha_1-1}(\frac{E_1}{L})y(\frac{E_1}{L})^\sigma = \left(\alpha_1\right)^{\alpha_1}\left(1-\alpha_1\right)^{1-\alpha_1}\left[\frac{\xi(K_0-E_1)}{L}\right]^{1-\alpha_2+\alpha_2\sigma}, \tag{55}$$

where $y(\cdot)$ is defined by (24). It uniquely determines $\frac{E_1}{L}$ because the two sides of (55) are strictly monotonic in $\frac{E_1}{L}$ but in opposite directions. To support such an equilibrium, we learn from Section 2 that the following is required:

$$\frac{E_1}{L} \in (k_1^*, k^*) \text{ and } \frac{\xi(K_0-E_1)}{L} \geq k_2^*.$$

Or equivalently,

$$\begin{aligned} \beta\xi A^{1-\sigma}\alpha_2\left(\Gamma(k^*)\right)^{\alpha_1-1}\left(y(k^*)\right)^\sigma &> \beta\xi A^{1-\sigma}\alpha_2\left(\Gamma(\frac{E_1}{L})\right)^{\alpha_1-1}\left(y(\frac{E_1}{L})\right)^\sigma \\ &> \beta\xi A^{1-\sigma}\alpha_2\left(\Gamma(k_1^*)\right)^{\alpha_1-1}\left(y(k_1^*)\right)^\sigma \\ \left(\alpha_1\right)^{\alpha_1}\left(1-\alpha_1\right)^{1-\alpha_1}\left(\frac{\xi(K_0-E_1)}{L}\right)^{1-\alpha_2+\alpha_2\sigma} &\geq \left(\alpha_1\right)^{\alpha_1}\left(1-\alpha_1\right)^{1-\alpha_1}(k_2^*)^{1-\alpha_2+\alpha_2\sigma} \end{aligned}$$

Observe that

$$\begin{aligned} \beta\xi A^{1-\sigma}\alpha_2\left(\Gamma(k_1^*)\right)^{\alpha_1-1}\left(y(k_1^*)\right)^\sigma &> \alpha_1^{\alpha_1}\left(1-\alpha_1\right)^{1-\alpha_1}(k_2^*)^{1-\alpha_2+\alpha_2\sigma} \\ &\Leftrightarrow \beta\xi > \left(\frac{1-\alpha_1}{1-\alpha_2}\right)^\sigma \end{aligned}$$

in which case,

$$\left(\alpha_1\right)^{\alpha_1}\left(1-\alpha_1\right)^{1-\alpha_1}\left(\frac{\xi(K_0-E_1)}{L}\right)^{1-\alpha_2+\alpha_2\sigma} > \beta\xi A^{1-\sigma}\alpha_2\left(\Gamma(k_1^*)\right)^{\alpha_1-1}\left(y_1(k_1^*)\right)^\sigma$$

$$\Updownarrow$$

$$\left(\frac{\alpha_1}{\alpha_2}\right)^{-1}\left[\beta\xi\left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{(1-\alpha_2)(1-\sigma)}\right]^{\frac{1}{1-\alpha_2+\alpha_2\sigma}}\frac{k_1^*}{\xi}+\frac{E_1}{L}<\frac{K_0}{L}.$$

It can be shown that

$$(\beta\xi)^{\frac{1}{1-\alpha_2+\sigma\alpha_2}}\frac{\left[\frac{\alpha_2(1-\alpha_1)}{\alpha_1\alpha_2-\alpha_1^2+\alpha_2-\alpha_2^2}\right]^{\frac{1-\alpha_1+\sigma\alpha_2}{(\alpha_2-\alpha_1)(1-\alpha_2+\sigma\alpha_2)}}}{\xi\left(\frac{\alpha_1}{\alpha_2}\right)\left(\frac{1-\alpha_2}{1-\alpha_1}\right)}k_1^*+\frac{E_1}{L}>\frac{K_0}{L},$$

so

$$(\beta\xi)^{\frac{1}{1-\alpha_2+\sigma\alpha_2}}\frac{\left[\frac{\alpha_2(1-\alpha_1)}{\left[\alpha_1\alpha_2-\alpha_1^2+\alpha_2-\alpha_2^2\right]}\right]^{\frac{1-\alpha_1+\sigma\alpha_2}{(\alpha_2-\alpha_1)(1-\alpha_2+\sigma\alpha_2)}}}{\xi\left(\frac{\alpha_1}{\alpha_2}\right)\left(\frac{1-\alpha_2}{1-\alpha_1}\right)}k_1^*+k^*>\frac{K_0}{L},$$

$$\left(\frac{\alpha_1}{\alpha_2}\right)^{-1}\left[\beta\xi\left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{(1-\alpha_2)(1-\sigma)}\right]^{\frac{1}{1-\alpha_2+\alpha_2\sigma}}\frac{k_1^*}{\xi}+k_1^*<\frac{K_0}{L}.$$

In particular, when $\sigma=1$, we have

$$\widetilde{\theta}_4\equiv\beta\frac{\alpha_2\left(1-\alpha_1\right)}{\left[\alpha_1\alpha_2-\alpha_1^2+\alpha_2-\alpha_2^2\right]}k^*+k^*>\frac{K_0}{L}>\widetilde{\theta}_0\equiv\frac{\alpha_2}{\alpha_1}\beta k_1^*+k_1^*.$$

The profit is given by

$$\frac{\Pi_{AB}}{W_1L}=\frac{\left(1-\alpha_1\right)}{\left(\alpha_2-\alpha_1\right)}\left[\Gamma(\frac{E_1}{L})\frac{E_1}{L}-\frac{\alpha_1}{1-\alpha_1}\right]\left[\left(\frac{\Gamma(\frac{E_1}{L})}{\psi}\right)^{\alpha_1-\alpha_2}-1\right],$$

where ψ is given by (8). The first-order condition is given by (22), replacing k with $\frac{E_1}{L}$. It can be rewritten as

$$\left(\frac{\Gamma(\frac{E_1}{L})}{\psi}\right)^{\alpha_1-\alpha_2}=\frac{1}{\left(1+\alpha_1-\alpha_2\right)+\frac{(\alpha_2-\alpha_1)\alpha_1}{(1-\alpha_1)\Gamma(\frac{E_1}{L})\frac{E_1}{L}}},$$

which determines $\frac{E_1}{L}$.

Now compare Π_{AB} with $\frac{\Pi_{CB}}{\widetilde{R}}$ (when $\sigma=1$), which is given by

$$\frac{\Pi_{CB}}{\widetilde{R}}=\beta\left(1-\left[A\left(\frac{K_0\beta\xi}{L(1+\beta)}\right)^{\alpha_2-\alpha_1}\right]^{-\frac{1}{1-\alpha_1}}\right)LW_1$$

when

$$\widetilde{\theta}_5\equiv\beta k^*+k^*\geq\frac{K_0}{L}\geq\widetilde{\theta}_0\equiv\left(\beta\frac{\alpha_2}{\alpha_1}+1\right)k_1^*.$$

In particular, when $\frac{K_0}{L}=\widetilde{\theta}_5$,

$$\begin{aligned}\frac{\Pi_{CB}}{\widetilde{R}}&=\beta\left(1-\left[A\left(k^*\beta\xi\right)^{\alpha_2-\alpha_1}\right]^{-\frac{1}{1-\alpha_1}}\right)LW_1\\&=\beta\left(1-\left[\frac{(\beta\xi)^{\alpha_2-\alpha_1}\alpha_1^{\alpha_1}\left(1-\alpha_1\right)}{\alpha_1\alpha_2-\alpha_1^2+\alpha_2-\alpha_2^2}\left(\frac{\alpha_2\left(1-\alpha_1\right)}{1-\alpha_2}\right)^{1-\alpha_1}\right]^{-\frac{1}{1-\alpha_1}}\right)LW_1\\&\geq\beta\left(1-\left[\left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{1+\alpha_2-2\alpha_1}\frac{\alpha_1^{\alpha_1}\alpha_2^{1-\alpha_1}\left(1-\alpha_1\right)}{\alpha_1\alpha_2-\alpha_1^2+\alpha_2-\alpha_2^2}\right]^{-\frac{1}{1-\alpha_1}}\right)LW_1\end{aligned}$$

When $\frac{K_0}{L}=\widetilde{\theta}_5$, we know that $\frac{E_1}{L}=k^*$ for pattern AB, (27) implies that

$$\Pi_{AB}=\frac{\left(\alpha_2-\alpha_1\right)^2}{\left[1-\alpha_2\right]\left[\alpha_1\alpha_2-\alpha_1^2+\alpha_2-\alpha_2^2\right]}W_1L.$$

Obviously, $\frac{\Pi_{CB}}{\widetilde{R}} \leq \Pi_{AB}$ when β is sufficiently small. When $\alpha_2 - \alpha_1 \to 0$, it turns out that $\frac{\Pi_{CB}}{\widetilde{R}} \to 0$ and $\Pi_{A1a} \to 0$. Let us see several numerical examples.

Suppose $\alpha_1 = \frac{1}{3}$ and $\alpha_2 = \frac{2}{3}$, then

$$\Pi_{AB} = W_1 L > \frac{\Pi_{CB}}{\widetilde{R}}.$$

Suppose $\alpha_1 = \frac{1}{2}$ and $\alpha_2 = 1$, then

$$\begin{aligned}\Pi_{AB} &= \frac{(\alpha_2 - \alpha_1)^2}{[1-\alpha_2]\,[\alpha_1\alpha_2 - \alpha_1^2 + \alpha_2 - \alpha_2^2]} W_1 L \to \infty,\\ \frac{\Pi_{CB}}{\widetilde{R}} &= \beta L W_1.\end{aligned}$$

Suppose $\alpha_1 = 0$ and $\alpha_2 = \frac{1}{2}$, then

$$\begin{aligned}\Pi_{AB} &= 2W_1 L,\\ \frac{\Pi_{CB}}{\widetilde{R}} &= \beta L W_1.\end{aligned}$$

Suppose $\alpha_1 = \frac{1}{4}$ and $\alpha_2 = \frac{1}{2}$, then

$$\begin{aligned}\Pi_{AB} &= \frac{2}{5} W_1 L,\\ \frac{\Pi_{CB}}{\widetilde{R}} &= \beta\left(1 - \frac{5}{9}\left[(\beta\xi)\left(\frac{3}{5}\right)\right]^{-\frac{1}{3}}\right) L W_1\\ &> \beta\left(1 - \frac{5}{9}\left[\frac{9}{10}\right]^{-\frac{1}{3}}\right) L W_1.\end{aligned}$$

Notice $\beta\xi > \frac{1-\alpha_1}{1-\alpha_2} = \frac{3}{2}$. So $\frac{\Pi_{CB}}{\widetilde{R}} > \Pi_{A1a}$ when $\beta > \frac{2}{5\left(1-\frac{5}{9}\left(\frac{9}{10}\right)^{-\frac{1}{3}}\right)} > \frac{9}{10}$.

In addition, whenever both Pattern AB and Pattern BB are feasible, the latter gives a strictly larger profit for firm M.

Pattern AA: both technologies in both periods

The analysis in Section 2.1 suggests that we must require $\frac{E_2}{L} \in (k_1^*, k_2^*)$. Applying Proposition 1, we have

$$C_2 = \alpha_1 k_1^{*\alpha_1 - 1} E_2 + (1-\alpha_1) k_1^{*\alpha_1} L.$$

Observe that

$$\frac{P_1}{R_1} = \frac{\left(\Gamma(\frac{E_1}{L})\right)^{\alpha_1 - 1}}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}};\ \frac{P_2}{R_2} = \frac{1}{\alpha_1 (k_1^*)^{\alpha_1 - 1}}.$$

(36) becomes

$$\beta\xi\left(\frac{a\xi(\frac{K_0}{L} - \frac{E_1}{L}) + b}{y_1(\frac{E_1}{L})}\right)^{-\sigma} = \left(\frac{1-\alpha_1}{\alpha_1} k_1^* \Gamma(\frac{E_1}{L})\right)^{1-\alpha_1}, \tag{56}$$

which uniquely determines $\frac{E_1}{L}$. The profit is $\pi_1(\left(\frac{E_1}{L}\right)^*) W_1 L$. To support this equilibrium, we require

$$\xi(\frac{K_0}{L} - \frac{E_1}{L}) \in (k_1^*, k_2^*) \text{ and } \frac{E_1}{L} \in (k_1^*, k^*),$$

which jointly imply

$$\left(\frac{(1-\alpha_1)}{1-\alpha_2}\right)^{-\sigma} \beta\xi k_1^{*-\sigma\alpha_1} < \left(\frac{1-\alpha_1}{\alpha_1} k_1^* \Gamma(\frac{E_1}{L})\right)^{1-\alpha_1} y_1(\frac{E_1}{L})^{-\sigma} < k_1^{*-\sigma\alpha_1},$$

which requires $\beta\xi < \left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{\sigma}$. In that case $k_1^* < \frac{E_1}{L} < \widetilde{k}_1^* (< k^*)$, where $\widetilde{k}_1^*$ is uniquely determined by

$$\left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{-\sigma}\left(\frac{1-\alpha_1}{\alpha_1}\right)^{-(1-\alpha_1)} \beta\xi k_1^{*-\sigma\alpha_1 - (1-\alpha_1)} = \Gamma(\widetilde{k}_1^*)^{1-\alpha_1} y_1(\widetilde{k}_1^*)^{-\sigma}.$$

On the other hand

$$\beta\xi\left(a\xi(\frac{K_0}{L} - \frac{E_1}{L}) + b\right)^{-\sigma} < k_1^{*-\sigma\alpha_1},$$

$$\frac{(\beta\xi)^{\frac{1}{\sigma}} - (1-\alpha_1)}{\alpha_1}\frac{k_1^*}{\xi} < \frac{K_0}{L} - \frac{E_1}{L} < \frac{k_2^*}{\xi}.$$

In summary, we must have

$$\frac{(\beta\xi)^{\frac{1}{\sigma}} - (1-\alpha_1)}{\alpha_1}\frac{k_1^*}{\xi} + k_1^* < \frac{K_0}{L} < \widetilde{k}_1^* + \frac{k_2^*}{\xi}.$$

In particular, when $\sigma = 1$, it requires

$$\beta\xi < \frac{1-\alpha_1}{1-\alpha_2},$$

and

$$\frac{\beta\xi - (1-\alpha_1)}{\alpha_1\xi}k_1^* + k_1^* < \frac{K_0}{L} < \frac{\alpha_2}{1-\alpha_2}\frac{(1-\alpha_1)\,k_1^*}{\alpha_1\xi} + \widetilde{k}_1^*.$$

Possibility B for Option 1 : only technology 2 in period 1

$$P_1/R_1 = \frac{\left(\frac{W_1}{R_1}\right)^{1-\alpha_1}}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}} = \frac{1}{\left(\frac{\alpha_2}{1-\alpha_2}\frac{L}{E_1}\right)^{1-\alpha_1}(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}},$$

because

$$\frac{R_1}{W_1} = \frac{\alpha_2}{1-\alpha_2}\frac{L}{E_1}.$$

The profit in period 1 is

$$\Pi_B = \frac{\left[\left(\frac{1}{k_2^*}\frac{E_1}{L}\right)^{\alpha_2-\alpha_1} - 1\right]}{(1-\alpha_2)}W_1 L.$$

Pattern BB: Only technology 2 in both periods

$$P_2/R_2 = \frac{\left(\frac{W_2}{R_2}\right)^{1-\alpha_2}}{A(\alpha_2)^{\alpha_2}(1-\alpha_2)^{1-\alpha_2}} = \frac{\left[\frac{\xi(K_0-E_1)}{L}\right]^{1-\alpha_2}}{A\alpha_2}.$$

Then (36) yields:

$$\beta\xi = \left[\frac{\xi(K_0-E_1)}{E_1}\right]^{1-\alpha_2+\sigma\alpha_2}\left(\frac{E_1}{L}\right)^{\alpha_1-\alpha_2}\frac{\alpha_1^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}}{A\alpha_2^{\alpha_1}(1-\alpha_2)^{1-\alpha_1}}. \tag{57}$$

To justify Pattern BB, we must have $E_1^* \geq k^*L$ and $E_2^* = \xi(K_0 - E_1) \geq k_2^*L$, or equivalently, $K_0 - \frac{k_2^*L}{\xi} \geq E_1 \geq k^*L$, thus $K_0 \geq \frac{k_2^*L}{\xi} + k^*L$which, by (57), are reduced to

$$k_2^{*1+\sigma\alpha_2-\alpha_1}\left(K_0 - \frac{k_2^*L}{\xi}\right)^{\alpha_1-(1+\sigma\alpha_2)} L^{1+\sigma\alpha_2-\alpha_1} \leq \beta\xi \leq \left[\frac{\xi(K_0-k^*L)}{k^*L}\right]^{1-\alpha_2+\sigma\alpha_2}.$$

So ultimately, we must require

$$\frac{(\beta\xi)^{\frac{1}{1+\sigma\alpha_2-\alpha_2}}k^*L}{\xi} + k^*L \leq K_0. \tag{58}$$

In particular, when $\sigma = 1$, (57) becomes

$$\left(\frac{E_1}{Lk_2^*}\right)^{\alpha_2-\alpha_1} = \frac{(K_0-E_1)}{\beta E_1}, \tag{59}$$

which uniquely determines E_1. Moreover, $\Pi_{BB} = \frac{\left[\left(\frac{1}{k_2^*}\frac{E_1}{L}\right)^{\alpha_2-\alpha_1}-1\right]}{(1-\alpha_2)}W_1L = \frac{\left[\frac{(K_0-E_1)}{\beta E_1}-1\right]}{(1-\alpha_2)}W_1L$. Whenever both Pattern AB and Pattern BB are feasible, the latter gives a strictly larger profit for firm M, so we can refine the previous Lemma can be further refined by adding that Pattern AB is an equilibrium only if $\frac{K_0}{L} \in (\widetilde{\theta}_0, \widetilde{\theta}_5)$.

F.2. Option 2: Technology 2 is first adopted in period 2

The market structure is perfectly competitive in period 1 with only technology 1. Thus

$$\frac{P_1}{R_1} = \frac{\left(\frac{R_1}{W_1}\right)^{\alpha_1-1}}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}},$$

$$\begin{aligned}
\frac{R_1}{W_1} &= \frac{\alpha_1}{1-\alpha_1}\frac{L}{E_1}, \\
P_1 &= \frac{R_1^{\alpha_1} W_1^{1-\alpha_1}}{\left(\alpha_1\right)^{\alpha_1}\left(1-\alpha_1\right)^{1-\alpha_1}}, \\
C_1 &= E_1^{\alpha_1} L^{1-\alpha_1}.
\end{aligned}$$

In period 2, either only technology 2 is operated, or both technologies are operated.

Pattern CB: only technology 1 in period 1 and only technology 2 in period 2

$$\begin{aligned}
P_2 &= \frac{R_2^{\alpha_1} W_2^{1-\alpha_1}}{\left(\alpha_1\right)^{\alpha_1}\left(1-\alpha_1\right)^{1-\alpha_1}}, \\
\frac{P_2}{R_2} &= \frac{\left(\frac{R_2}{W_2}\right)^{\alpha_1-1}}{\left(\alpha_1\right)^{\alpha_1}\left(1-\alpha_1\right)^{1-\alpha_1}}, \\
\frac{R_2}{W_2} &= \frac{\alpha_2}{\left(1-\alpha_2\right)\left(\frac{E_2}{L}\right)}, \\
C_2 &= A\left[\xi(K_0-E_1)\right]^{\alpha_2} L^{1-\alpha_2},
\end{aligned}$$

(36) implies

$$\beta\xi\left(A\left(\frac{E_1}{L}\right)^{\alpha_2-\alpha_1}\right)^{-\sigma} = \left(\frac{\xi(K_0-E_1)}{E_1}\right)^{-\alpha_1+1+\sigma\alpha_2}\left(\frac{\alpha_2\left(1-\alpha_1\right)}{\left(1-\alpha_2\right)\alpha_1}\right)^{\alpha_1-1},$$

which uniquely determines E_1^*. Since $\xi\frac{K_0-E_1}{L} \geq k^*$,

$$\frac{E_1}{L} \geq \left[\frac{k^{*-\alpha_1+1+\sigma\alpha_2}}{\beta\xi A^{-\sigma}}\left(\frac{\alpha_2\left(1-\alpha_1\right)}{\left(1-\alpha_2\right)\alpha_1}\right)^{\alpha_1-1}\right]^{\frac{1}{-\alpha_1+1+\sigma\alpha_1}}.$$

No conditions need to be imposed for period 1 because technology 2 is privately accessible. To justify that firm M serves the whole market in period 2 with technology 2, we require $E_2^* \geq k^* L$, which means $K_0 - \frac{k^*}{\xi}L \geq E_1$, therefore

$$\frac{E_1}{L} \leq \left(\frac{1}{A}\right)^{\frac{1}{(\alpha_2-\alpha_1)}}\left[\frac{\left(\frac{\alpha_2(1-\alpha_1)}{(1-\alpha_2)\alpha_1}\right)^{\alpha_1-1}}{\beta\xi\left(k^*\right)^{\alpha_1-1-\sigma\alpha_2}}\right]^{-\frac{1}{\sigma(\alpha_2-\alpha_1)}},$$

$$K_0 \geq \left[\frac{A^{\sigma} k^{*1+\sigma\alpha_2-\alpha_1}}{\beta\xi}\left(\frac{\alpha_2\left(1-\alpha_1\right)}{\left(1-\alpha_2\right)\alpha_1}\right)^{\alpha_1-1}\right]^{\frac{1}{\sigma\alpha_1-\alpha_1+1}} L + \frac{k^*}{\xi}L, \tag{60}$$

$$\begin{aligned}
\widetilde{R} &= \frac{\xi R_2}{R_1} = \frac{\xi\alpha_2\left(1-\alpha_1\right)}{\alpha_1\frac{E_2}{E_1}\left(1-\alpha_2\right)}\frac{W_2}{W_1}, \\
\Pi_{CB} &= \frac{\left[\left(\frac{1}{k_2^*}\frac{E_2}{L}\right)^{\alpha_2-\alpha_1}-1\right]}{\left(1-\alpha_2\right)} W_2 L, \\
\frac{\Pi_{CB}}{\widetilde{R}} &= \frac{\alpha_1\frac{E_2}{E_1}\left[\left(\frac{1}{k_2^*}\frac{E_2}{L}\right)^{\alpha_2-\alpha_1}-1\right] W_1 L}{\xi\alpha_2\left(1-\alpha_1\right)}.
\end{aligned}$$

In addition, we have

$$\frac{\partial E_1^*}{\partial\beta} < 0; \frac{\partial E_1^*}{\partial A} \geq 0; \frac{\partial E_1^*}{\partial L} \leq 0; \frac{\partial E_1^*}{\partial\xi} \geq 0; \frac{\partial E_1^*}{\partial K_0} > 0,$$

where "=" holds if and only if $\sigma = 1$, in which case the Euler equation becomes

$$\beta\xi\left(\xi(K_0-E_1)\right)^{\alpha_1-1-\alpha_2} = AE_1^{-1}\left(\frac{1}{L}\right)^{\alpha_2-\alpha_1}\left(\frac{\alpha_2\left(1-\alpha_1\right)}{\left(1-\alpha_2\right)\alpha_1}\right)^{\alpha_1-1},$$

or equivalently,

$$E_1 = E_2^{-\alpha_1+\alpha_2+1} \frac{A}{\beta\xi} \left(\frac{1}{L}\right)^{\alpha_2-\alpha_1} \left(\frac{\alpha_2(1-\alpha_1)}{(1-\alpha_2)\alpha_1}\right)^{\alpha_1-1}.$$

Thus

$$\begin{aligned}\frac{\Pi_{CB}}{\widetilde{R}} &= \frac{\alpha_1 \frac{E_2}{E_1}\left[\left(\frac{1}{k_2^*}\frac{E_2}{L}\right)^{\alpha_2-\alpha_1} - 1\right] W_1 L}{\xi\alpha_2(1-\alpha_1)} \\ &= \frac{\beta\left(\frac{\alpha_2(1-\alpha_1)}{(1-\alpha_2)\alpha_1}\right)^{-\alpha_1}\left[\left(\frac{1}{k_2^*}\frac{E_2}{L}\right)^{\alpha_2-\alpha_1} - 1\right] W_1 L}{A(1-\alpha_2)\left(\frac{E_2}{L}\right)^{\alpha_2-\alpha_1}}.\end{aligned}$$

Recall under BB: $K_0 \geq \frac{(\beta\xi)^{\frac{1}{1+\sigma\alpha_2-\alpha_2}} k^* L}{\xi} + k^* L$. Under CB,

$$K_0 \geq \left[\frac{\left[\frac{(1-\alpha_1)\alpha_2}{\alpha_1\alpha_2-\alpha_1^2+\alpha_2-\alpha_2^2}\left(\frac{1-\alpha_1}{1-\alpha_2}\right)\right]^{\sigma}}{\beta\xi}\right]^{\frac{1}{\sigma\alpha_1-\alpha_1+1}} \frac{\alpha_1(1-\alpha_2)}{\alpha_2(1-\alpha_1)} k^* L + \frac{k^*}{\xi} L,$$

$$\text{when } \sigma = 1,\ \frac{K_0}{L} \geq \widetilde{\theta}_2 \equiv \frac{\alpha_1(1-\alpha_1)}{\alpha_1\alpha_2-\alpha_1^2+\alpha_2-\alpha_2^2}\frac{k^*}{\beta\xi} + \frac{k^*}{\xi},$$

$$\frac{\Pi_{CB}}{\widetilde{R}} = \frac{\alpha_1 \frac{E_2}{E_1}\left[\left(\frac{1}{k_2^*}\frac{E_2}{L}\right)^{\alpha_2-\alpha_1} - 1\right] W_1 L}{\xi\alpha_2(1-\alpha_1)}.$$

Now we can compare $\frac{\Pi_{CB}}{\widetilde{R}}$ with Π_{B1}. In particular, when $\sigma = 1$, we have $\frac{\Pi_{CB}}{\widetilde{R}} > \Pi_{B1a} \Leftrightarrow$

$$\frac{\beta\left(\frac{\alpha_2(1-\alpha_1)}{(1-\alpha_2)\alpha_1}\right)^{-\alpha_1}\left[\left(\frac{1}{k_2^*}\right)^{\alpha_2-\alpha_1} - \left(\frac{E_2}{L}\right)^{-\alpha_2+\alpha_1}\right]}{A} > \frac{(K_0 - E_1)}{\beta E_1} - 1.$$

Note that LHS is smaller than $\frac{\beta\left(\frac{\alpha_2(1-\alpha_1)}{(1-\alpha_2)\alpha_1}\right)^{-\alpha_1}\left(\frac{1}{k_2^*}\right)^{\alpha_2-\alpha_1}}{A}$, but for RHS (B1a),

$$\frac{(K_0 - E_1)}{\beta E_1} - 1 = \frac{\beta\left(\frac{\alpha_2(1-\alpha_1)}{(1-\alpha_2)\alpha_1}\right)^{-\alpha_1}\left(\frac{1}{k_2^*}\right)^{\alpha_2-\alpha_1}}{A}$$

if and only if

$$\begin{aligned}E_1 &= \left[1 + \frac{(1-\alpha_2)\beta}{1-\alpha_1}\right]^{\frac{1}{\alpha_2-\alpha_1}} k_2^* L, \\ K_0 &= \left[\left(1 + \frac{(1-\alpha_2)\beta}{1-\alpha_1}\right)\beta + 1\right]\left[1 + \frac{(1-\alpha_2)\beta}{1-\alpha_1}\right]^{\frac{1}{\alpha_2-\alpha_1}} k_2^* L.\end{aligned}$$

In other words, we know that $\frac{\Pi_{CB}}{\widetilde{R}} < \Pi_{B1a}$ when

$$K_0 \geq \left[\left(1 + \frac{(1-\alpha_2)\beta}{1-\alpha_1}\right)\beta + 1\right]\left[1 + \frac{(1-\alpha_2)\beta}{1-\alpha_1}\right]^{\frac{1}{\alpha_2-\alpha_1}} k_2^* L,$$

F.3. Option 3 (c): Technology 2 is never adopted

This occurs only when $\frac{K_0}{L}$ is sufficiently small so that the market cannot support technology 2 in either period. The threshold value is derived when analyzing Pattern CA.

Appendix G. Proof of Propositions 9 and 10

Option 2: Technology 2 is first adopted in Period 2.

The market structure is perfectly competitive in period 1 with only technology 1 in operation. Thus

$$\begin{aligned}
\frac{P_1}{R_1} &= \frac{\left(\frac{R_1}{W_1}\right)^{\alpha_1-1}}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}},\\
\frac{R_1}{W_1} &= \frac{\alpha_1}{1-\alpha_1}\frac{L}{E_1},\\
P_1 &= \frac{(R_1)^{\alpha_1}W_1^{1-\alpha_1}}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}},\\
C_1 &= E_1^{\alpha_1}L^{1-\alpha_1}.
\end{aligned}$$

In period 2, there are two possibilities: Possibility A is that only technology 2 is operated in period 2. Possibility B is that both technologies are operated in period 2.

Pattern CB: only technology 1 in period 1 and only technology 2 in period 2

$$\begin{aligned}
P_2 &= \frac{(R_2)^{\alpha_1}W_2^{1-\alpha_1}}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}},\\
\frac{P_2}{R_2} &= \frac{\left(\frac{R_2}{W_2}\right)^{\alpha_1-1}}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}},\\
\frac{R_2}{W_2} &= \frac{\alpha_1}{1-\alpha_1}A^{\frac{1}{1-\alpha_1}}\left(\frac{E_2}{L}\right)^{\frac{1-\alpha_2}{\alpha_1-1}},\\
C_2 &= A\left[\xi(K_0-E_1)\right]^{\alpha_2}L^{1-\alpha_2}.
\end{aligned}$$

(36) implies

$$\beta\xi A^{1-\sigma}\left(\frac{E_1}{L}\right)^{1-\alpha_1+\sigma\alpha_1} = (\xi\frac{K_0-E_1}{L})^{1-\alpha_2+\sigma\alpha_2},$$

which uniquely determines E_1^*. No conditions need to be imposed for technology 1 to be implemented in period 1 when technology 2 is privately accessible. To justify that firm M can serve the whole market in period 2 with technology 2, we require $E_2^* \geq k^*L$, which means

$$\xi\frac{K_0-E_1}{L} \geq k^*;\ \frac{E_1}{L} \geq \left[\frac{k^{*1-\alpha_2+\sigma\alpha_2}}{\beta\xi A^{1-\sigma}}\right]^{\frac{1}{1-\alpha_1+\sigma\alpha_1}},$$

therefore

$$\frac{K_0}{L} \geq \frac{k^*}{\xi} + \left[\frac{k^{*1-\alpha_2+\sigma\alpha_2}}{\beta\xi A^{1-\sigma}}\right]^{\frac{1}{1-\alpha_1+\sigma\alpha_1}}. \tag{61}$$

In addition, we have

$$\frac{\partial E_1^*}{\partial\beta}<0;\ \frac{\partial E_1^*}{\partial A}\leq 0;\ \frac{\partial E_1^*}{\partial L}\geq 0;\ \frac{\partial E_1^*}{\partial\xi}\leq 0;\ \frac{\partial E_1^*}{\partial K_0}>0,$$

where "=" holds if and only if $\sigma = 1$, in which case

$$\begin{aligned}
E_1^* &= \frac{K_0}{1+\beta};\ E_2^* = \frac{\beta\xi K_0}{1+\beta};\\
\frac{R_1}{W_1} &= \frac{\alpha_1(1+\beta)}{(1-\alpha_1)}\frac{L}{K_0};\ \frac{R_2}{W_2} = \frac{\alpha_1}{1-\alpha_1}A^{\frac{1}{1-\alpha_1}}\left(\frac{\beta\xi K_0}{L(1+\beta)}\right)^{\frac{1-\alpha_2}{\alpha_1-1}};\\
\Pi_{CB} &= \left(A^{\frac{1}{1-\alpha_1}}\left(\frac{\beta\xi K_0}{(1+\beta)L}\right)^{\frac{\alpha_2-\alpha_1}{1-\alpha_1}}-1\right)LW_2;\\
\widetilde{R} &= \frac{\xi^{\frac{\alpha_2-\alpha_1}{1-\alpha_1}}A^{\frac{1}{1-\alpha_1}}\left(\frac{K_0}{L}\right)^{\frac{\alpha_2-\alpha_1}{1-\alpha_1}}W_2}{(1+\beta)^{\frac{\alpha_2-\alpha_1}{1-\alpha_1}}\beta^{\frac{1-\alpha_2}{1-\alpha_1}}W_1};\\
\frac{\Pi_{CB}}{\widetilde{R}} &= \beta\left(1-\left[A\left(\frac{K_0\beta\xi}{L(1+\beta)}\right)^{\alpha_2-\alpha_1}\right]^{-\frac{1}{1-\alpha_1}}\right)LW_1,
\end{aligned}$$

and (61) becomes

$$\frac{K_0}{L} \geq \frac{k^*}{\xi} + \frac{k^*}{\beta\xi}.$$

For more general $\sigma \in [0,1)$, we have

$$\frac{\Pi_{CB}}{\widetilde{R}} = \left[\frac{L^{\frac{\alpha_1-\alpha_2}{\alpha_1-1}}}{\xi^{\frac{\alpha_1-\alpha_2}{\alpha_1-1}} E_1^* \left[(K_0 - E_1^*)\right]^{\frac{1-\alpha_2}{\alpha_1-1}}} \left(\left(\frac{\beta\xi K_0}{(1+\beta) L} \right)^{\frac{\alpha_2-\alpha_1}{1-\alpha_1}} - A^{-\frac{1}{1-\alpha_1}} \right) \right] L W_1.$$

Now we can compare $\frac{\Pi_{CB}}{\widetilde{R}}$ with the profit in possibility B for option 1, Π_{B1}. In particular, when $\sigma = 1$, we have

$$\frac{\Pi_{CB}}{\widetilde{R}} > \Pi_{BB} \Leftrightarrow$$

$$\beta + 1 - \left(A \left[\frac{1}{\beta\frac{\alpha_2}{\alpha_1} + 1} \right]^{\alpha_2-\alpha_1} \right)^{\frac{1}{1-\alpha_1}} \left(K_0/L\right)^{\frac{\alpha_2-\alpha_1}{1-\alpha_1}}$$

$$> \beta \left[A \left(\frac{\beta\xi}{(1+\beta)} \right)^{\alpha_2-\alpha_1} \right]^{-\frac{1}{1-\alpha_1}} \left(K_0/L\right)^{-\frac{\alpha_2-\alpha_1}{1-\alpha_1}},$$

which holds if and only if

$$x_1 < \left(K_0/L\right)^{\frac{\alpha_2-\alpha_1}{1-\alpha_1}} < x_2, \tag{62}$$

where

$$x_{1,2} \equiv \frac{(1+\beta) \mp \sqrt{(1+\beta)^2 - 4\beta \left(\frac{(1+\beta)}{\beta\xi\left[\beta\frac{\alpha_2}{\alpha_1}+1\right]} \right)^{\frac{\alpha_2-\alpha_1}{1-\alpha_1}}}}{2\left(A \left[\frac{1}{\beta\frac{\alpha_2}{\alpha_1}+1} \right]^{\alpha_2-\alpha_1} \right)^{\frac{1}{1-\alpha_1}}},$$

$$\Delta \equiv (1+\beta)^2 - 4\beta \left(\frac{(1+\beta)}{\beta\xi\left[\beta\frac{\alpha_2}{\alpha_1} + 1\right]} \right)^{\frac{\alpha_2-\alpha_1}{1-\alpha_1}} > 0.$$

Therefore, when (62) holds, $\frac{\frac{\Pi_{CB}}{\widetilde{R}}}{\Pi_{BB}} > 1$; otherwise $\frac{\frac{\Pi_{CB}}{\widetilde{R}}}{\Pi_{BB}} \leq 1$. Pattern CB requires $\frac{K_0}{L} \geq \widetilde{\theta}_1 \equiv \frac{(1+\beta)}{\beta\xi} k^*$.

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